Lp theory for the interaction between the incompressible Navier-Stokes system and a damped plate

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Abstract

We consider a viscous incompressible fluid governed by the Navier-Stokes system written in a domain where a part of the boundary can deform. We assume that the corresponding displacement follows a damped beam equation. Our main results are the existence and uniqueness of strong solutions for the corresponding fluid-structure interaction system in an L^p-L^q setting for small times or for small data. An important ingredient of the proof consists in the study of a linear parabolic system coupling the non stationary Stokes system and a damped plate equation. We show that this linear system possesses the maximal regularity property by proving the $\mathcal{R}$-sectoriality of the corresponding operator. The proof of the main results is then obtained by an appropriate change of variables to handle the free boundary and a fixed point argument to treat the nonlinearities of this system.

Key words. Incompressible Navier-Stokes System, Fluid-structure interaction, Strong solutions, Maximal L^p regularity.
AMS subject classifications. 35Q35, 76D03, 76D05, 74F10.

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1 Introduction

In this work, we study the interaction between a viscous incompressible fluid and a deformable structure located on a part of the fluid domain boundary. More precisely, we denote by $\mathcal{F}$ the reference domain for the fluid.

We assume that it is a smooth bounded domain of $\mathbb{R}^3$ such that its boundary $\partial\mathcal{F}$ contains a flat part Γ_S corresponding to the reference domain of the plate. We assume $\Gamma_S = \mathcal{S} \times \{0\}$, where $\mathcal{S}$ is a smooth domain of $\mathbb{R}^2$ and we set $\Gamma_0 := \partial\mathcal{F} \setminus \overline{\Gamma_S}$. The set Γ_0 is rigid and remains unchanged whereas the plate domain Γ_S can deform through exterior forces and in particular the force coming from the fluid and if we denote by η is displacement, then the plate domain changes from Γ_S to

$$\Gamma_S(\eta) := \{(s, \eta(s))\ ;\ s \in \mathcal{S}\}\,.$$

In our study, we consider only displacements η regular enough and satisfying the boundary conditions (the plate is clamped):

$$\eta = \nabla_s \eta \cdot n_{\mathcal{S}} = 0 \quad \text{on} \quad \partial\mathcal{S} \tag{1.1}$$

and a condition insuring that the deformed plate does not have any contact with the other part of the boundary of the fluid domain:

$$\Gamma_0 \cap \Gamma_S(\eta) = \emptyset. \tag{1.2}$$

We have denoted by $n_{\mathcal{S}}$ the unitary exterior normal to $\partial\mathcal{S}$ and in the whole article we add the index s in the gradient and in the Laplace operators if they apply to functions defined on $\mathcal{S} \subset \mathbb{R}^2$ (and we keep the usual notation for functions defined on a domain of $\mathbb{R}^3$).

With the above notations and hypotheses, $\Gamma_0 \cup \overline{\Gamma_S(\eta)}$ corresponds to a closed simple and regular surface which interior is the fluid domain $\mathcal{F}(\eta)$. In what follows, we consider that η is also a function of time and its evolution is governed by a plate equation. If $\eta(t,\cdot)$ satisfies the above conditions, we can define the fluid domain $\mathcal{F}(\eta(t))$ and we then denote by $(\widetilde{v}, \widetilde{\pi})$ the Eulerian velocity and the pressure of the fluid and we assume that they satisfy the incompressible Navier-Stokes system in $\mathcal{F}(\eta(t))$. Then the corresponding system we analyze reads as follows:

$$\begin{cases} \partial_t \widetilde{v} + (\widetilde{v} \cdot \nabla)\widetilde{v} - \operatorname{div} \mathbb{T}(\widetilde{v}, \widetilde{\pi}) = 0, & t > 0, x \in \mathcal{F}(\eta(t)), \\ \operatorname{div} \widetilde{v} = 0 & t > 0, x \in \mathcal{F}(\eta(t)), \\ \widetilde{v}(t, s, \eta(t,s)) = \partial_t \eta(t,s) e_3 & t > 0, s \in \mathcal{S}, \\ \widetilde{v} = 0 & t > 0, x \in \Gamma_0, \\ \partial_{tt}\eta + \alpha\Delta_s^2 \eta - \beta\Delta_s \eta - \gamma\Delta_s \partial_t \eta = \mathbb{H}(\widetilde{v}, \widetilde{\pi}, \eta) & t > 0, s \in \mathcal{S}, \\ \eta = \nabla_s \eta \cdot n_{\mathcal{S}} = 0 & t > 0, s \in \partial\mathcal{S}, \end{cases} \tag{1.3}$$

where (e_1, e_3, e_3) is the canonical basis of $\mathbb{R}^3$. The fluid stress tensor $\mathbb{T}(\widetilde{v}, \widetilde{\pi})$ is given by

$$\mathbb{T}(\widetilde{v}, \widetilde{\pi}) = 2\nu D(\widetilde{v}) - \widetilde{\pi} I_3, \quad D(\widetilde{v}) = \frac{1}{2}\left(\nabla\widetilde{v} + \nabla\widetilde{v}^\top\right). \tag{1.4}$$

The function $\mathbb{H}$ corresponds to the force of the fluid acting on the plate and can be expressed as follows:

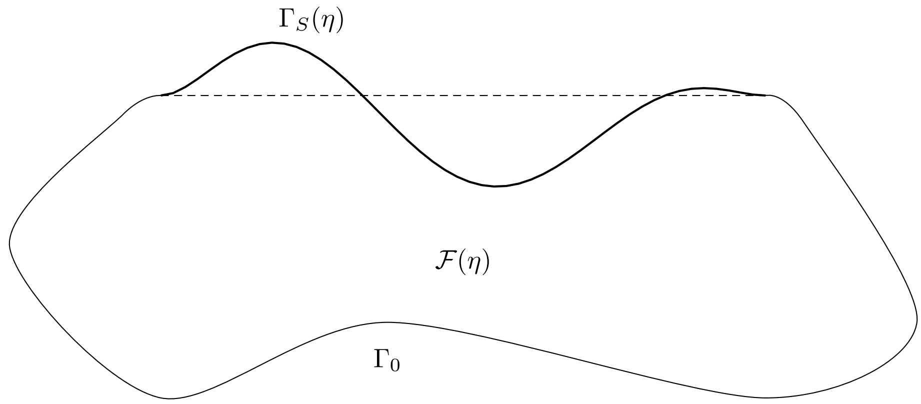
$$\mathbb{H}(\widetilde{v}, \widetilde{\pi}, \eta) = -\sqrt{1 + |\nabla_s \eta|^2}\, (\mathbb{T}(\widetilde{v}, \widetilde{\pi})\widetilde{n})\,|_{\Gamma_S(\eta(t))} \cdot e_3, \tag{1.5}$$

where

$$\widetilde{n} = \frac{1}{\sqrt{1 + |\nabla_s \eta|^2}} \left[-\nabla_s \eta, 1\right]^\top,$$

is the unit normal to $\Gamma_S(\eta(t))$ outward $\mathcal{F}(\eta(t))$. The above system is completed by the following initial data

$$\eta(0,\cdot) = \eta_1^0 \text{ in } \mathcal{S}, \quad \partial_t \eta(0,\cdot) = \eta_2^0 \text{ in } \mathcal{S}, \quad \widetilde{v}(0,\cdot) = \widetilde{v}^0 \text{ in } \mathcal{F}(\eta_1^0). \tag{1.6}$$



System (1.3) is a simplified model for blood flow in arteries (see, for instance the survey article [38]) and α, β, γ are non negative constants that corresponds to the physical properties of the wall tissue. Our analysis will be done in the case $\alpha > 0$, $\beta \geqslant 0$ and $\gamma > 0$ and to simplify, we consider in what follows the case

$$\alpha = 1, \quad \beta = 0, \quad \gamma = 1,$$

and the other cases can be done in the same way. Let us remark that the term $-\gamma \Delta_s \partial_t \eta$ corresponds to the damping in the plate equation. The other positive constant, appearing in (1.4) is the viscosity ν.

An important remark in the study of (1.3)-(1.6) is that a solution $(\widetilde{v}, \widetilde{\pi}, \eta)$ satisfies

$$0 = \int_{\mathcal{F}(\eta(t))} \operatorname{div} \widetilde{v} \, dx = \int_{\Gamma_S(\eta(t))} \widetilde{v} \cdot \widetilde{n} \, d\Gamma = \frac{d}{dt} \int_{\mathcal{S}} \eta \, ds.$$

Assuming that η_1^0 has a zero mean, we deduce that this property is preserved for η all along. This leads us to consider the space

$$L^q_m(\mathcal{S}) = \left\{ f \in L^q(\mathcal{S}) \; ; \; \int_{\mathcal{S}} f \, ds = 0 \right\}, \tag{1.7}$$

and the orthogonal projection $P_m : L^q(\mathcal{S}) \to L^q_m(\mathcal{S})$, that is

$$P_m f = f - \frac{1}{|\mathcal{S}|} \int_{\mathcal{S}} f \, ds \qquad (f \in L^q(\mathcal{S})). \tag{1.8}$$

Taking the projection of the plate equation in (1.3) onto $L^q_m(\mathcal{S})$ and onto $L^q_m(\mathcal{S})^\perp$ yields the following two equations:

$$\partial_{tt} \eta + P_m \Delta_s^2 \eta - \Delta_s \partial_t \eta = P_m \left(\mathbb{H}(\widetilde{v}, \widetilde{\pi}, \eta) \right) \qquad t > 0, s \in \mathcal{S}, \tag{1.9}$$

and

$$\int_{\mathcal{S}} \widetilde{\pi}(t, s, \eta(t, s)) \, ds = \int_{\mathcal{S}} \Delta_s^2 \eta(t, s) \, ds + \int_{\mathcal{S}} \sqrt{1 + |\nabla_s \eta|^2} \left[(2\nu D\widetilde{v}) \widetilde{n} \right] (t, s, \eta(t, s)) \cdot e_3 \, ds. \tag{1.10}$$

This means that, in contrast to the Navier-Stokes system without structure, the pressure is not determined up to a constant. In what follows, we only keep (1.9) and solve the corresponding system up to constant for the pressure, and equation (1.10) is used at the end to fix the constant for the pressure. We thus consider the following system

$$\begin{cases} \partial_t \widetilde{v} + (\widetilde{v} \cdot \nabla) \widetilde{v} - \operatorname{div} \mathbb{T}(\widetilde{v}, \widetilde{\pi}) = 0 & t > 0, x \in \mathcal{F}(\eta(t)), \\ \operatorname{div} \widetilde{v} = 0 & t > 0, x \in \mathcal{F}(\eta(t)), \\ \widetilde{v}(t, s, \eta(t, s)) = \partial_t \eta(t, s) e_3 & t > 0, s \in \mathcal{S}, \\ \widetilde{v} = 0 & t > 0, x \in \Gamma_0, \\ \partial_{tt} \eta + P_m \Delta_s^2 \eta - \Delta_s \partial_t \eta = P_m \mathbb{H}(\widetilde{v}, \widetilde{\pi}, \eta) & t > 0, s \in \mathcal{S}, \\ \eta = \nabla_s \eta \cdot n_{\mathcal{S}} = 0 & t > 0, s \in \partial \mathcal{S}, \\ \eta(0, \cdot) = \eta_1^0 \text{ in } \mathcal{S}, \quad \partial_t \eta(0, \cdot) = \eta_2^0 \text{ in } \mathcal{S}, \quad \widetilde{v}(0, \cdot) = \widetilde{v}^0 \text{ in } \mathcal{F}(\eta_1^0). \end{cases} \tag{1.11}$$

To state our main result, we introduce some notations for our functional spaces. Firstly $W^{s,q}(\Omega)$, with $s \geqslant 0$ and $q \geqslant 1$, denotes the usual Sobolev space. Let $k, k' \in \mathbb{N}$, $k < k'$. For $1 \leqslant p < \infty$, $1 \leqslant q < \infty$, we consider the standard definition of the Besov spaces by real interpolation of Sobolev spaces

$$B^s_{q,p}(\mathcal{F}) = \Big(W^{k,q}(\mathcal{F}), W^{k',q}(\mathcal{F})\Big)_{\theta,p} \text{ where } s = (1-\theta)k + \theta k', \quad \theta \in (0,1).$$

We refer to [1] and [45] for a detailed presentation of the Besov spaces. We also introduce functional spaces for the fluid velocity and pressure for a spatial domain depending on the displacement η of the structure. Let $1 < p, q < \infty$ and $\eta \in L^p(0,\infty; W^{4,q}(\mathcal{S})) \cap W^{2,p}(0,\infty; L^q(\mathcal{S}))$ satisfying (1.1) and (1.2). We show in Section 2 that there exists a mapping $X = X_\eta$ such that $X(t,\cdot)$ is a C^1-diffeomorphism from $\mathcal{F}$ onto $\mathcal{F}(\eta(t))$ and such that $X \in L^p(0,\infty; W^{2,q}(\mathcal{F})) \cap W^{2,p}(0,\infty; L^q(\mathcal{F}))$. Then for $T \in (0,\infty]$, we define

$$\begin{aligned}
&L^p(0,T; L^q(\mathcal{F}(\eta(\cdot)))) := \left\{v \circ X^{-1} \ ; \ v \in L^p(0,T; L^q(\mathcal{F}))\right\},\\
&L^p(0,T; W^{2,q}(\mathcal{F}(\eta(\cdot)))) := \left\{v \circ X^{-1} \ ; \ v \in L^p(0,T; W^{2,q}(\mathcal{F}))\right\},\\
&W^{1,p}(0,T; L^q(\mathcal{F}(\eta(\cdot)))) := \left\{v \circ X^{-1} \ ; \ v \in W^{1,p}(0,T; L^q(\mathcal{F}))\right\},\\
&C^0([0,T]; W^{1,q}(\mathcal{F}(\eta(\cdot)))) := \left\{v \circ X^{-1} \ ; \ v \in C^0([0,T]; W^{1,q}(\mathcal{F}))\right\},\\
&C^0([0,T]; B^{2(1-1/p)}_{q,p}(\mathcal{F}(\eta(\cdot)))) := \left\{v \circ X^{-1} \ ; \ v \in C^0([0,T]; B^{2(1-1/p)}_{q,p}(\mathcal{F}))\right\},
\end{aligned}$$

where we have set $(v \circ X^{-1})(t,x) := v(t, (X(t,\cdot))^{-1}(x))$ for simplicity.

Finally, let us give the conditions we need on the initial conditions for the system (1.11): we assume

$$\eta^0_1 \in B^{2(2-1/p)}_{q,p}(\mathcal{S}), \quad \eta^0_2 \in B^{2(1-1/p)}_{q,p}(\mathcal{S}), \quad \widetilde{v}^0 \in B^{2(1-1/p)}_{q,p}(\mathcal{F}(\eta^0_1)) \tag{1.12}$$

with the compatibility conditions

$$\eta^0_1 = \nabla_s \eta^0_1 \cdot n_{\mathcal{S}} = 0 \quad \text{on } \partial\mathcal{S}, \quad \Gamma_0 \cap \Gamma_S(\eta^0_1) = \emptyset, \quad \int_{\mathcal{S}} \eta^0_1 \, ds = 0, \quad \int_{\mathcal{S}} \eta^0_2 \, ds = 0, \quad \operatorname{div} \widetilde{v}^0 = 0 \quad \text{in } \mathcal{F}(\eta^0_1), \tag{1.13}$$

and

$$\begin{cases}
\widetilde{v}^0(s, \eta^0_1(s)) \cdot \widetilde{n}^0 = \eta^0_2(s) e_3 \cdot \widetilde{n}^0 \quad s \in \mathcal{S}, \quad \widetilde{v}^0 \cdot \widetilde{n}^0 = 0 \quad \text{on } \Gamma_0 & \text{if} \quad \dfrac{1}{p} + \dfrac{1}{2q} > 1,\\
\widetilde{v}^0(s, \eta^0_1(s)) = \eta^0_2(s) e_3 \quad s \in \mathcal{S}, \quad \widetilde{v}^0 = 0 \quad \text{on } \Gamma_0, \quad \eta^0_2 = 0 \quad \text{on } \partial\mathcal{S} & \text{if} \quad \dfrac{1}{p} + \dfrac{1}{2q} < 1,\\
\nabla_s \eta^0_2 \cdot n_S = 0 \text{ on } \partial\mathcal{S} & \text{if} \quad \dfrac{1}{p} + \dfrac{1}{2q} < \dfrac{1}{2}.
\end{cases} \tag{1.14}$$

Here $\widetilde{n}^0$ is the unit exterior normal to $\Gamma_S(\eta^0_1)$ outward $\mathcal{F}(\eta^0_1)$.

We are now in a position to state our main results. The first one is the local in time existence and uniqueness of strong solutions for (1.11).

Theorem 1.1. *Let $p, q \in (1,\infty)$ such that*

$$\frac{1}{p} + \frac{1}{2q} \neq 1, \quad \frac{1}{p} + \frac{1}{2q} \neq \frac{1}{2} \quad \textit{and} \quad \frac{1}{p} + \frac{3}{2q} < \frac{3}{2}. \tag{1.15}$$

Let us assume that $\eta^0_1 = 0$ and $(\eta^0_2, \widetilde{v}^0)$ satisfies (1.12), (1.13), (1.14). Then there exists $T > 0$, depending only on $(\eta^0_2, \widetilde{v}^0)$, such that the system (1.11) admits a unique strong solution $(\widetilde{v}, \widetilde{\pi}, \eta)$ in the class of functions satisfying

$$\begin{gathered}
\widetilde{v} \in L^p(0,T; W^{2,q}(\mathcal{F}(\eta(\cdot)))) \cap L^\infty(0,T; B^{2(1-1/p)}_{q,p}(\mathcal{F}(\eta(\cdot)))) \cap W^{1,p}(0,T; L^q(\mathcal{F}(\eta(\cdot)))),\\
\widetilde{\pi} \in L^p(0,T; W^{1,q}_m(\mathcal{F}(\eta(\cdot)))),\\
\eta \in L^p(0,T; W^{4,q}(\mathcal{S})) \cap L^\infty(0,T; B^{2(2-1/p)}_{q,p}(\mathcal{S})) \cap W^{1,p}(0,T; W^{2,q}(\mathcal{S})),\\
\partial_t \eta \in L^p(0,T; W^{2,q}(\mathcal{S})) \cap L^\infty(0,T; B^{2(1-1/p)}_{q,p}(\mathcal{S})) \cap W^{1,p}(0,T; L^q(\mathcal{S})).
\end{gathered}$$

Moreover, $\Gamma_0 \cap \Gamma_S(\eta(t)) = \emptyset$ for all $t \in [0,T]$.

Our second main result asserts the global existence and uniqueness of strong solution for (1.11) under a smallness condition on the initial data.

Theorem 1.2. *Let $p,q \in (1,\infty)$ satisfying the conditions (1.15). Then there exists $\beta_0 > 0$ such that, for all $\beta \in [0,\beta_0]$ there exist ε_0 and $C > 0$, such that for any $(\eta_1^0, \eta_2^0, \widetilde{v}^0)$ satisfying (1.12), (1.13), (1.14) and*

$$\|\widetilde{v}^0\|_{B^{2(1-1/p)}_{q,p}(\mathcal{F}(\eta_1^0))} + \|\eta_1^0\|_{B^{2(2-1/p)}_{q,p}(\mathcal{S})} + \|\eta_2^0\|_{B^{2(1-1/p)}_{q,p}(\mathcal{S})} < \varepsilon_0, \tag{1.16}$$

the system (1.11) admits a unique strong solution $(\widetilde{v}, \widetilde{\pi}, \eta)$ in the class of functions satisfying

$$\begin{gathered}
\widetilde{v} \in L^p_\beta(0,\infty; W^{2,q}(\mathcal{F}(\eta(\cdot)))) \cap L^\infty_\beta(0,\infty; B^{2(1-1/p)}_{q,p}(\mathcal{F}(\eta(\cdot)))) \cap W^{1,p}_\beta(0,\infty; L^q(\mathcal{F}(\eta(\cdot)))),\\
\widetilde{\pi} \in L^p_\beta(0,\infty; W^{1,q}_m(\mathcal{F}(\eta(\cdot)))),\\
\eta \in L^p_\beta(0,\infty; W^{4,q}(\mathcal{S})) \cap L^\infty_\beta(0,\infty; B^{2(2-1/p)}_{q,p}(\mathcal{S})) \cap W^{1,p}_\beta(0,\infty; W^{2,q}(\mathcal{S})),\\
\partial_t \eta \in L^p_\beta(0,\infty; W^{2,q}(\mathcal{S})) \cap L^\infty_\beta(0,\infty; B^{2(1-1/p)}_{q,p}(\mathcal{S})) \cap W^{1,p}_\beta(0,\infty; L^q(\mathcal{S})).
\end{gathered}$$

Moreover, $\Gamma_0 \cap \Gamma_S(\eta(t)) = \emptyset$ for all $t \in [0,\infty)$.

In the above statement, we have used a similar notation as in (1.7):

$$L^q_m(\mathcal{F}) := \left\{ f \in L^q(\mathcal{F}) \ ; \ \int_{\mathcal{F}} f = 0 \ dx \right\}, \quad W^{s,q}_m(\mathcal{F}) := W^{s,q}(\mathcal{F}) \cap L^q_m(\mathcal{F}).$$

We also set

$$W^{s,q}_m(\mathcal{S}) = W^{s,q}(\mathcal{S}) \cap L^q_m(\mathcal{S}).$$

We denote by $W^{s,q}_0(\mathcal{S})$ the closure of $C^\infty_c(\mathcal{S})$ in $W^{s,q}(\mathcal{S})$ and we set

$$W^{s,q}_{0,m}(\mathcal{S}) = W^{s,q}_0(\Omega) \cap L^q_m(\mathcal{S}).$$

We define similarly $W^{s,q}_0(\mathcal{F})$, $W^{s,q}_{0,m}(\mathcal{F})$.

Finally, we also need the following notation in what follows: for $T \in (0,\infty]$,

$$\begin{gathered}
W^{1,2}_{p,q}((0,T);\mathcal{F}) = L^p(0,T; W^{2,q}(\mathcal{F})) \cap W^{1,p}(0,T; L^q(\mathcal{F})),\\
W^{2,4}_{p,q}((0,T);\mathcal{S}) = L^p(0,T; W^{4,q}(\mathcal{S})) \cap W^{1,p}(0,T; W^{2,q}(\mathcal{S})) \cap W^{2,p}(0,T; L^q(\mathcal{S})),\\
W^{1,2}_{p,q}((0,T);\mathcal{S}) = L^p(0,T; W^{2,q}(\mathcal{S})) \cap W^{1,p}(0,T; L^q(\mathcal{S})).
\end{gathered}$$

We have the following embeddings (see, for instance, [2, Theorem 4.10.2, p.180]),

$$W^{1,2}_{p,q}((0,T);\mathcal{F}) \hookrightarrow C^0_b([0,T); B^{2(1-1/p)}_{q,p}(\mathcal{F})), \tag{1.17}$$

$$W^{2,4}_{p,q}((0,T);\mathcal{S}) \hookrightarrow C^0_b([0,T); B^{2(2-1/p)}_{q,p}(\mathcal{S})) \cap C^1_b([0,T); B^{2(1-1/p)}_{q,p}(\mathcal{S})) \tag{1.18}$$

where C^k_b is the set of continuous and bounded functions with derivatives continuous and bounded up to the order k. In particular, in what follows, we use the following norm for $W^{1,2}_{p,q}((0,T);\mathcal{F})$:

$$\|f\|_{W^{1,2}_{p,q}((0,T);\mathcal{F})} := \|f\|_{L^p(0,T;W^{2,q}(\mathcal{F}))} + \|f\|_{W^{1,p}(0,T;L^q(\mathcal{F}))} + \|f\|_{C^0_b([0,T);B^{2(1-1/p)}_{q,p}(\mathcal{F}))}$$

and we proceed similarly for the two other spaces.

For $\beta \geqslant 0$, $p \in [1,\infty]$ and for $\mathbb{X}$ a Banach space, we also introduce the notation

$$L^p_\beta(0,\infty;\mathbb{X}) := \left\{ f \ ; \ t \mapsto e^{\beta t} f(t) \in L^p(0,\infty;\mathbb{X}) \right\},$$

and a similar notation for $W^{1,2}_{p,q,\beta}((0,\infty);\mathcal{F})$, $W^{2,4}_{p,q,\beta}((0,\infty);\mathcal{S})$, etc.

Let us give some remarks on Theorem 1.1 and Theorem 1.2. First let us point out that the system (1.11) has already been studied by several authors: existence of weak solutions ([9], [26], [37]), uniqueness of weak solutions ([25]), existence of strong solutions ([7], [32], [34]), feedback stabilization ([40], [5]), global existence of strong solutions and study of the contacts ([22]). Some works consider also the case of a beam/plate without damping (that is without the term $-\Delta_s \partial_t \eta$): [21], [23], [6]. We refer, for instance, to [24] and references therein for a concise description of recent progress in this field. It is important to notice that all the above works correspond to a "Hilbert" framework whereas our results are done in a "L^p-L^q" framework. Working in such a framework allows us to extend the result obtained in the "Hilbert" framework, but it should be noticed that several questions on fluid-structure interaction systems, in the "Hilbert" framework, have been handled by considering a "L^p-L^q" framework: for instance, the uniqueness of weak solutions (see [20], [8]), the asymptotic behavior for large time (see [16]), the asymptotic behavior for small structures (see [31]), etc.

For this approach, several recent results have been obtained for fluid systems, with or without structure. For instance, one can quote [19] (viscous incompressible fluid), [15], (viscous compressible fluid), [28], [27] (viscous compressible fluid with rigid bodies), [18], [35] (incompressible viscous fluid and rigid bodies). Here we consider an incompressible viscous fluid coupled with a structure satisfying an infinite-dimensional system and we thus need to go beyond the theory developed for instance in [35].

Our approach to prove Theorem 1.1 and Theorem 1.2 is quite classical. Since the fluid domain $\mathcal{F}(\eta(t))$ depends on the structure displacement η, we first reformulate the problem in a fixed domain. This is achieved by "geometric" change of variables. Next we associate the original nonlinear problem to a linear one. The linear system preserves the fluid-structure coupling. A crucial step here is to establish the L^p-L^q regularity property in the infinite time horizon. This is done by showing the associate linear operator $\mathcal{R}$-sectorial and generates an exponentially stable semigroup. We then use the Banach fixed point theorem to prove existence and uniqueness results. Note that for Theorem 1.2, we assume the same conditions on (p,q) than for Theorem 1.1 but the result should be also true for $\frac{1}{p}+\frac{3}{2q}=\frac{3}{2}$. However to deal with this case one needs some precise results on the interpolation of Besov spaces (see for instance Lemma 2.1).

Let us also remark that this work could also be done in the corresponding 2D/1D model, that is $\mathcal{F}$ a regular bounded domain in $\mathbb{R}^2$ such that $\partial\mathcal{F}$ contains a flat part $\Gamma_S = \mathcal{S}\times\{0\}$, where $\mathcal{S}$ is an open bounded interval of $\mathbb{R}$. In that case, we would obtain the same result as in Theorem 1.1 and in Theorem 1.2 but with the following condition on p,q:

$$\frac{1}{p}+\frac{1}{2q}\neq 1, \quad \frac{1}{p}+\frac{1}{2q}\neq\frac{1}{2} \quad \text{and} \quad \frac{1}{p}+\frac{1}{q}<\frac{3}{2}.$$

The plan of the paper is as follows. In the next section, we use a change of variables to rewrite the governing equations in a cylindrical domain and we also restate our result after change of variables. Then, in Section 3, we recall several important results about maximal L^p regularity for Cauchy problems and in particular how to use the $\mathcal{R}$-sectoriality property. We use these results to study in Section 4 the linearized system. Finally in Section 5 and in Section 6, we estimate the nonlinear terms which allows us to prove the main results with a fixed point argument.

2 Change of Variables

In order to prove Theorem 1.2, we first rewrite the system (1.11) in the cylindrical domain $(0,\infty)\times\mathcal{F}$ by constructing an invertible mapping $X(t,\cdot)$ from the reference configuration $\mathcal{F}$ onto $\mathcal{F}(\eta(t))$. More generally, for any $\eta\in C^1(\overline{\mathcal{S}})$ satisfying (1.1) and a smallness condition

$$\|\eta\|_{L^\infty(\mathcal{S})}\leqslant c_0 \tag{2.1}$$

that ensures in particular (1.2), we can construct a diffeomorphism $X_\eta:\mathcal{F}\to\mathcal{F}(\eta)$. To do this, we follow the approach of [5]: there exists $\alpha>0$ such that

$$\mathcal{V}_{-\alpha} := \mathcal{S}\times(-\alpha,0)\subset\mathcal{F}, \quad \mathcal{V}_\alpha := \mathcal{S}\times(0,\alpha)\subset\mathbb{R}^3\setminus\mathcal{F}. \tag{2.2}$$

Notice that, $\partial \mathcal{V}_\alpha \cap \partial \mathcal{F} = \Gamma_S$. We consider $\psi \in C_c^\infty(\mathbb{R})$ such that

$$\psi = 1 \text{ in } (-\alpha/2, \alpha/2), \quad \psi = 0 \text{ in } \mathbb{R} \setminus (-\alpha, \alpha), \quad 0 \leqslant \psi \leqslant 1. \tag{2.3}$$

Let us extend η by 0 in $\mathbb{R} \setminus \mathcal{S}$ so that $\eta \in C_c^1(\mathbb{R})$ and let us define X_η by

$$X_\eta \left(\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \right) = \left(\begin{bmatrix} y_1 \\ y_2 \\ y_3 + \psi(y_3)\eta(y_1, y_2) \end{bmatrix} \right) \quad \left(y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \in \mathbb{R}^3 \right). \tag{2.4}$$

If we choose c_0 in (2.1) as

$$c_0 := \frac{1}{2\|\psi'\|_{L^\infty(\mathbb{R})}} \tag{2.5}$$

then X_η is a C^1-diffeomorphism from $\mathcal{F}$ onto $\mathcal{F}(\eta)$ with $X_\eta(\Gamma_S) = \Gamma_S(\eta)$. Note that (2.1) and (2.5) yield that $|\eta| \leqslant \alpha/2$ in $\mathcal{S}$.

Let us assume now that η depends also on time and satisfies for all t relation (2.1) with c_0 given by (2.5). We can define

$$X(t, \cdot) := X_{\eta(t)}. \tag{2.6}$$

In particular, $X(t, \cdot)$ is a C^1-diffeomorphism from $\mathcal{F}$ onto $\mathcal{F}(\eta(t))$. For each $t \geqslant 0$, we denote by $Y(t, \cdot) = X(t, \cdot)^{-1}$, the inverse of $X(t, \cdot)$. We have $X \in C_b^0([0, \infty); C^1(\overline{\mathcal{F}}))$ and for all $t \in (0, \infty)$, $y = [y_1 \ y_2 \ y_3]^\top \in \mathcal{S} \times (-\alpha/2, \alpha/2)$,

$$\det \nabla X(t, y) = 1, \qquad \operatorname{Cof}(\nabla X)(t, y) = \begin{bmatrix} 1 & 0 & -\partial_{y_1}\eta(t, y_1, y_2) \\ 0 & 1 & -\partial_{y_2}\eta(t, y_1, y_2) \\ 0 & 0 & 1 \end{bmatrix}. \tag{2.7}$$

We consider the following change of unknowns

$$v(t, y) = \operatorname{Cof} \nabla X^\top(t, y)\widetilde{v}(t, X(t, y)), \qquad \pi(t, y) = \widetilde{\pi}(t, X(t, y)), \qquad (t, y) \in (0, \infty) \times \mathcal{F}. \tag{2.8}$$

The system (1.11) can be rewritten in the form

$$\begin{cases} \partial_t v - \operatorname{div} \mathbb{T}(v, \pi) = F(v, \pi, \eta) & t > 0, y \in \mathcal{F}, \\ \operatorname{div} v = 0 & t > 0, y \in \mathcal{F}, \\ v(t, s, 0) = \partial_t \eta(t, s) e_3 & t > 0, s \in \mathcal{S}, \\ v = 0 & t > 0, y \in \Gamma_0, \\ \partial_{tt}\eta + P_m\left(\Delta_s^2 \eta\right) - \Delta_s \partial_t \eta & \\ \qquad = -P_m\Big(\mathbb{T}(v, \pi)|_{\Gamma_S} e_3 \cdot e_3\Big) + P_m\Big(H(v, \pi, \eta)\Big) & t > 0, s \in \mathcal{S}, \\ \eta = \nabla_s \eta \cdot n_{\mathcal{S}} = 0 & t > 0, s \in \partial\mathcal{S}, \\ \eta(0, \cdot) = \eta_1^0 \text{ in } \mathcal{S}, \quad \partial_t \eta(0, \cdot) = \eta_2^0 \text{ in } \mathcal{S}, \quad v(0, \cdot) = v^0 \text{ in } \mathcal{F}, & \end{cases} \tag{2.9}$$

where

$$v^0(y) := \operatorname{Cof} \nabla X^\top(0, y)\widetilde{v}^0(X(0, y)) = \operatorname{Cof} \nabla X_{\eta_1^0}^\top(y)\widetilde{v}^0(X_{\eta_1^0}(y)). \tag{2.10}$$

Let us write

$$a := \operatorname{Cof}(\nabla Y)^\top, \quad b := \operatorname{Cof}(\nabla X)^\top \tag{2.11}$$

so that

$$v(t, y) = b(t, y)\widetilde{v}(t, X(t, y)), \quad \widetilde{v}(t, x) = a(t, x)v(t, Y(t, x)). \tag{2.12}$$

After some standard calculation, we find that in (2.9), the expressions of F and H are

$$\begin{aligned}
F_\alpha(v,\pi,\eta) = \nu \sum_{i,j,k} b_{\alpha i} \frac{\partial^2 a_{ik}}{\partial x_j^2}(X) v_k + 2\nu \sum_{i,j,k,\ell} b_{\alpha i} \frac{\partial a_{ik}}{\partial x_j}(X) \frac{\partial v_k}{\partial y_\ell} \frac{\partial Y_\ell}{\partial x_j}(X) \\
+ \nu \sum_{j,\ell,m} \frac{\partial^2 v_\alpha}{\partial y_\ell \partial y_m} \left(\frac{\partial Y_\ell}{\partial x_j}(X) \frac{\partial Y_m}{\partial x_j}(X) - \delta_{\ell,j}\delta_{m,j} \right) + \nu \sum_{j,\ell} \frac{\partial v_\alpha}{\partial y_\ell} \frac{\partial^2 Y_\ell}{\partial x_j^2}(X) \\
- \sum_{k,i} \frac{\partial \pi}{\partial y_k} \left(\det(\nabla X) \frac{\partial Y_\alpha}{\partial x_i}(X) \frac{\partial Y_k}{\partial x_i}(X) - \delta_{\alpha,i}\delta_{k,i} \right) \\
- \sum_{i,j,k,m} b_{\alpha i} \frac{\partial a_{ik}}{\partial x_j}(X) a_{jm}(X) v_k v_m - \frac{1}{\det(\nabla X)} \left[(v \cdot \nabla) v \right]_\alpha \\
- \left[b(\partial_t a)(X) v \right]_\alpha - \left[(\nabla v)(\partial_t Y)(X) \right]_\alpha, \quad (2.13)
\end{aligned}$$

$$\begin{aligned}
H(v,\pi,\eta) = \nu \Bigg\{ \sum_{k=1}^3 \left(\sum_{i=1}^2 \partial_{s_i} \eta \left[\frac{\partial a_{ik}}{\partial x_3}(X) + \frac{\partial a_{3k}}{\partial x_i}(X) \right] - 2 \frac{\partial a_{3k}}{\partial x_3}(X) \right) v_k \\
+ \sum_{k=1}^3 \left(\sum_{i=1}^2 \partial_{s_i} \eta \left[a_{ik}(X) \frac{\partial Y_\ell}{\partial x_3}(X) + a_{3k}(X) \frac{\partial Y_\ell}{\partial x_i}(X) \right] - 2 \left[a_{3k}(X) \frac{\partial Y_\ell}{\partial x_3}(X) - \delta_{2,k}\delta_{2,\ell} \right] \right) \frac{\partial v_k}{\partial y_\ell} \Bigg\} (t,s,1). \quad (2.14)
\end{aligned}$$

We prove the following result

Lemma 2.1. *Let $1 < p, q < \infty$ such that*

$$\frac{1}{p} + \frac{3}{2q} < \frac{3}{2}, \tag{2.15}$$

and $(\eta_1^0, \widetilde{v}^0)$ satisfies (1.12). Then v^0 defined by (2.10) satisfies $v^0 \in B_{q,p}^{2(1-1/p)}(\mathcal{F})$.

Proof. By using (2.15), we deduce that $\eta_1^0 \in C^1(\overline{\mathcal{S}})$. In particular, the map

$$\widetilde{v}^0 \mapsto \widehat{v}^0 = \widetilde{v}^0 \circ X_{\eta_1^0} \tag{2.16}$$

is linear and continuous from $L^q(\mathcal{F}(\eta_1^0))$ into $L^q(\mathcal{F})$. Let us show that it is also continuous from $W^{2,q}(\mathcal{F}(\eta_1^0))$ into $W^{2,q}(\mathcal{F})$: Some computation yields

$$\frac{\partial^2 \widehat{v}^0}{\partial y_i \partial y_j}(y) = \sum_{k,\ell} \frac{\partial^2 \widetilde{v}^0}{\partial x_\ell \partial x_k}(X_{\eta_1^0}(y)) \frac{\partial X_{\eta_1^0,\ell}}{\partial y_j}(y) \frac{\partial X_{\eta_1^0,k}}{\partial y_i}(y) + \frac{\partial \widetilde{v}^0}{\partial x_k}(X_{\eta_1^0}(y)) \frac{\partial^2 X_{\eta_1^0,k}}{\partial y_i \partial y_j}(y). \tag{2.17}$$

Using that $\eta_1^0 \in C^1(\overline{\mathcal{S}})$, we deduce that the first term in the right-hand side of the above relation belongs to $L^q(\mathcal{F})$. For the second term, we first note that $\frac{\partial \widetilde{v}^0}{\partial x_k}(X(\cdot)) \in W^{1,q}(\mathcal{F})$ and $\frac{\partial^2 X_{\eta_1^0,k}}{\partial y_i \partial y_j} \in B_{q,p}^{2(1-1/p)}(\mathcal{F})$. Therefore by [45, Theorem(i), page 196], $\frac{\partial^2 X_{\eta_1^0,k}}{\partial y_i \partial y_j} \in W^{s_1,q}(\mathcal{F})$ for any $s_1 < 2(1-1/p)$. Applying standard result on the product of Sobolev spaces we conclude that the second term in (2.17) also belongs to $L^q(\mathcal{F})$.

Then by interpolation, we deduce that the map (2.16) is linear continuous from $B_{q,p}^{2(1-1/p)}(\mathcal{F}(\eta_1^0))$ into $B_{q,p}^{2(1-1/p)}(\mathcal{F})$. Therefore, if $v^0 \in B_{q,p}^{2(1-1/p)}(\mathcal{F}(\eta_1^0))$, we have

$$\operatorname{Cof} \nabla X_{\eta_1^0}^\top \in B_{q,p}^{1+2(1-1/p)}(\mathcal{F}), \quad \widehat{v}^0 \in B_{q,p}^{2(1-1/p)}(\mathcal{F})$$

and we deduce that the product $v^0 \in B_{q,p}^{2(1-1/p)}(\mathcal{F})$ by using [42, Theorem 2, pp.191-192, relation (17)]. $\square$

Using the above lemma and the definition of X defined in (2.6), the hypotheses (1.12), (1.13), (1.14) on the initial conditions are transformed into the following conditions:

$$\eta_1^0 \in B^{2(2-1/p)}_{q,p}(\mathcal{S}), \quad \eta_2^0 \in B^{2(1-1/p)}_{q,p}(\mathcal{S}), \quad v^0 \in B^{2(1-1/p)}_{q,p}(\mathcal{F}), \tag{2.18}$$

$$\eta_1^0 = \nabla_s \eta_1^0 \cdot n_{\mathcal{S}} = 0 \quad \text{on } \partial\mathcal{S}, \quad \Gamma_0 \cap \Gamma_S(\eta_1^0) = \emptyset, \quad \int_{\mathcal{S}} \eta_1^0 \, ds = 0, \quad \int_{\mathcal{S}} \eta_2^0 \, ds = 0,$$
$$\operatorname{div}(v^0) = 0 \quad \text{in } \mathcal{F}, \tag{2.19}$$

$$\begin{cases} v^0(s,0)\cdot e_3 = \eta_2^0(s) \quad s \in \mathcal{S}, \quad v^0 \cdot n = 0 \quad \text{on } \Gamma_0 & \text{if} \quad \dfrac{1}{p} + \dfrac{1}{2q} > 1, \\ v^0(s,0) = \eta_2^0(s) e_3 \quad s \in \mathcal{S}, \quad v^0 = 0 \quad \text{on } \Gamma_0, \quad \eta_2^0 = 0 \quad \text{on } \partial\mathcal{S} & \text{if} \quad \dfrac{1}{p} + \dfrac{1}{2q} < 1, \\ \nabla_s \eta_2^0 \cdot n_S = 0 \text{ on } \partial\mathcal{S} & \text{if} \quad \dfrac{1}{p} + \dfrac{1}{2q} < \dfrac{1}{2}. \end{cases} \tag{2.20}$$

Here n is the unit normal to $\partial\mathcal{F}$ outward $\mathcal{F}$ and in particular on Γ_S, $n = e_3$.

Using the above change of variables Theorem 1.1 and Theorem 1.2 can be rephrased as

Theorem 2.2. *Let $p, q \in (1,\infty)$ satisfying the condition (1.15). Let us assume that $\eta_1^0 = 0$ and (η_2^0, v^0) satisfies (2.18), (2.19), (2.20). Then there exists $T > 0$, depending only on (η_2^0, v^0), such that the system (2.9) admits a unique strong solution (v, π, η) in the class of functions satisfying*

$$v \in W^{1,2}_{p,q}((0,T);\mathcal{F}), \quad \pi \in L^p(0,T; W^{1,q}_m(\mathcal{F})), \quad \eta \in W^{2,4}_{p,q}((0,T);\mathcal{S})$$

Moreover, η satisfies (2.1) and $X(t,\cdot) : \mathcal{F} \to \mathcal{F}(\eta(t))$ is a C^1-diffeomorphism for all $t \in [0,T]$.

Theorem 2.3. *Let $p, q \in (1,\infty)$ satisfying the condition (1.15). Then there exists $\beta_0 > 0$ such that, for all $\beta \in [0, \beta_0]$, there exist ε_0 and $C > 0$, such that for any $(\eta_1^0, \eta_2^0, v^0)$ satisfying (2.18), (2.19), (2.20) and*

$$\|\eta_1^0\|_{B^{2(2-1/p)}_{q,p}(\mathcal{S})} + \|\eta_2^0\|_{B^{2(1-1/p)}_{q,p}(\mathcal{S})} + \|v^0\|_{B^{2(1-1/p)}_{q,p}(\mathcal{F})} < \varepsilon_0, \tag{2.21}$$

the system (2.9) admits a unique strong solution (v, π, η) in the class of functions satisfying

$$v \in W^{1,2}_{p,q,\beta}((0,\infty);\mathcal{F}), \quad \pi \in L^p_\beta(0,\infty; W^{1,q}_m(\mathcal{F})), \quad \eta \in W^{2,4}_{p,q,\beta}((0,\infty);\mathcal{S})$$

Moreover, η satisfies (2.1) and $X(t,\cdot) : \mathcal{F} \to \mathcal{F}(\eta(t))$ is a C^1-diffeomorphism for all $t \in [0,\infty)$.

3 Some Background on $\mathcal{R}$-sectorial Operators

In this section, we recall some important facts on $\mathcal{R}$-sectorial operators. This notion is associated with the property of $\mathcal{R}$-boundedness ($\mathcal{R}$ for Randomized) for a family of operators that we recall here (see, for instance, [46, 10, 11, 30]):

Definition 3.1. *Let $\mathcal{X}$ and $\mathcal{Y}$ be Banach spaces. A family of operators $\mathcal{E} \subset \mathcal{L}(\mathcal{X},\mathcal{Y})$ is called $\mathcal{R}-$bounded if there exist $p \in [1,\infty)$ and a constant $C > 0$, such that for any integer $N \geqslant 1$, any $T_1, \dots T_N \in \mathcal{E}$, any independent Rademacher random variables $r_1, \dots, r_N$, and any $x_1, \dots, x_N \in \mathcal{X}$,*

$$\left(\mathbb{E} \left\| \sum_{j=1}^N r_j T_j x_j \right\|_{\mathcal{Y}}^p \right)^{1/p} \leqslant C \left(\mathbb{E} \left\| \sum_{j=1}^N r_j x_j \right\|_{\mathcal{X}}^p \right)^{1/p}.$$

The smallest constant C in the above inequality is called the $\mathcal{R}_p$-bound of $\mathcal{E}$ on $\mathcal{L}(\mathcal{X},\mathcal{Y})$ and is denoted by $\mathcal{R}_p(\mathcal{E})$.

In the above definition, we denote by $\mathbb{E}$ the expectation and a Rademacher random variable is a symmetric random variables with value in $\{-1,1\}$. It is proved in [11, p.26] that this definition is independent of $p \in [1,\infty)$.

We have the following useful properties (see Proposition 3.4 in [11]):

$$\mathcal{R}_p(\mathcal{E}_1+\mathcal{E}_2) \leqslant \mathcal{R}_p(\mathcal{E}_1)+\mathcal{R}_p(\mathcal{E}_2), \quad \mathcal{R}_p(\mathcal{E}_1\mathcal{E}_2) \leqslant \mathcal{R}_p(\mathcal{E}_1)\mathcal{R}_p(\mathcal{E}_2). \tag{3.1}$$

For any $\beta \in (0,\pi)$, we write

$$\Sigma_\beta = \{\lambda \in \mathbb{C}\setminus\{0\} \ ; \ |\arg(\lambda)| < \beta\}.$$

We recall the following definition:

Definition 3.2 (sectorial and $\mathcal{R}$-sectorial operators)**.** *Let A be a densely defined closed linear operator on a Banach space $\mathcal{X}$ with domain $\mathcal{D}(A)$. We say that A is a ($\mathcal{R}$)-sectorial operator of angle $\beta \in (0,\pi)$ if*

$$\Sigma_\beta \subset \rho(A)$$

and if the set

$$R_\beta = \left\{\lambda(\lambda-A)^{-1} \ ; \ \lambda \in \Sigma_\beta\right\}$$

is ($\mathcal{R}$)-bounded in $\mathcal{L}(\mathcal{X})$.

We denote by $M_\beta(A)$ (respectively $\mathcal{R}_\beta(A)$) the bound (respectively the $\mathcal{R}$-bound) of R_β. One can replace in the above definitions R_β by the set

$$\widetilde{R_\beta} = \left\{A(\lambda-A)^{-1} \ ; \ \lambda \in \Sigma_\beta\right\}.$$

In that case, we denote the uniform bound and the $\mathcal{R}$-bound by $\widetilde{M_\beta}(A)$ and $\widetilde{\mathcal{R}_\beta}(A)$.

This notion of $\mathcal{R}$-sectorial operators is related to the maximal regularity of type L^p by the following result due to [46] (see also [11, p.45]).

Theorem 3.3. *Let $\mathcal{X}$ be a UMD Banach space and A a densely defined, closed linear operator on $\mathcal{X}$. Then the following assertions are equivalent:*

1. *For any $T \in (0,\infty]$ and for any $f \in L^p(0,T;\mathcal{X})$, the Cauchy problem*

$$u' = Au + f \quad \text{in} \quad (0,T), \quad u(0) = 0 \tag{3.2}$$

admits a unique solution u with $u', Au \in L^p(0,T;\mathcal{X})$ and there exists a constant $C>0$ such that

$$\|u'\|_{L^p(0,T;\mathcal{X})} + \|Au\|_{L^p(0,T;\mathcal{X})} \leqslant C\|f\|_{L^p(0,T;\mathcal{X})}.$$

2. *A is $\mathcal{R}$-sectorial of angle $> \frac{\pi}{2}$.*

We recall that $\mathcal{X}$ is a UMD Banach space if the Hilbert transform is bounded in $L^p(\mathbb{R};\mathcal{X})$ for $p \in (1,\infty)$. In particular, the closed subspaces of $L^q(\Omega)$ for $q \in (1,\infty)$ are UMD Banach spaces. We refer the reader to [2, pp.141–147] for more information on UMD spaces.

Combining the above theorem with [13, Theorem 2.4] and [44, Theorem 1.8.2], we can deduce the following result on the system

$$u' = Au + f \quad \text{in} \quad (0,\infty), \quad u(0) = u_0. \tag{3.3}$$

Corollary 3.4. *Let $\mathcal{X}$ be a UMD Banach space, $1<p<\infty$ and let A be a closed, densely defined operator in $\mathcal{X}$ with domain $\mathcal{D}(A)$. Let us assume that A is a $\mathcal{R}$-sectorial operator of angle $> \frac{\pi}{2}$ and that the semigroup generated by A has negative exponential type. Then for every $u_0 \in (\mathcal{X},\mathcal{D}(A))_{1-1/p,p}$ and for every $f \in L^p(0,\infty;\mathcal{X})$, the system (3.3) admits a unique solution in $L^p(0,\infty;\mathcal{D}(A)) \cap W^{1,p}(0,\infty;\mathcal{X})$.*

Let us also mention, the following useful result on the perturbation theory of $\mathcal{R}$-sectoriality, obtained in [29, Corollary 2].

Proposition 3.5. *Let A be a $\mathcal{R}$-sectorial operator of angle β on a Banach space $\mathcal{X}$. Let $B : \mathcal{D}(B) \to \mathcal{X}$ be a linear operator such that $\mathcal{D}(A) \subset \mathcal{D}(B)$ and such that there exist $a, b \geqslant 0$ satisfying*

$$\|Bx\|_{\mathcal{X}} \leqslant a\|Ax\|_{\mathcal{X}} + b\|x\|_{\mathcal{X}} \quad (x \in \mathcal{D}(A)). \tag{3.4}$$

If

$$a < \frac{1}{\widetilde{M}_\beta(A)\widetilde{\mathcal{R}}_\beta(A)} \quad \text{and} \quad \lambda > \frac{bM_\beta(A)\widetilde{\mathcal{R}}_\beta(A)}{1 - a\widetilde{M}_\beta(A)\widetilde{\mathcal{R}}_\beta(A)},$$

then $A + B - \lambda$ is $\mathcal{R}$-sectorial of angle β.

4 Linearized System

In order to study the system (2.9), we linearized it and use the theory of the previous section. To this aim, we introduce the operator $\mathcal{T} : L^2(\mathcal{S}) \to L^2(\partial\mathcal{F})$ defined by

$$\begin{aligned} &(\mathcal{T}\eta)(y) = (P_m\eta(s))\, e_3 \quad \text{if} \quad y = (s, 0) \in \Gamma_S, \\ &(\mathcal{T}\eta)(y) = 0 \qquad\qquad \text{if} \quad y \in \Gamma_0. \end{aligned} \tag{4.1}$$

We consider the following linear system

$$\begin{cases} \partial_t v - \operatorname{div} \mathbb{T}(v, \pi) = f & \text{in } (0, \infty) \times \mathcal{F}, \\ \operatorname{div} v = 0 & \text{in } (0, \infty) \times \mathcal{F}, \\ v = \mathcal{T}\eta_2 & \text{on } (0, \infty) \times \partial\mathcal{F} \\ \partial_t \eta_1 = \eta_2 & \text{in } (0, \infty) \times \mathcal{S}, \\ \partial_t \eta_2 + P_m\left(\Delta_s^2 \eta_1\right) - \Delta_s \eta_2 = -P_m\Big(\mathbb{T}(v, \pi)|_{\Gamma_S} e_3 \cdot e_3\Big) + P_m h & \text{in } (0, \infty) \times \mathcal{S}, \\ \eta_1 = \nabla_s \eta_1 \cdot n_{\mathcal{S}} = 0 & \text{on } (0, \infty) \times \partial\mathcal{S}, \\ \eta_1(0, \cdot) = \eta_1^0 \text{ in } \mathcal{S}, \quad \eta_2(0, \cdot) = \eta_2^0 \text{ in } \mathcal{S}, \quad v(0, \cdot) = v^0 \text{ in } \mathcal{F}. \end{cases} \tag{4.2}$$

One can simplify the system (4.2): using that $\operatorname{div} v = 0$ in $\mathcal{F}$ and $v_1 = v_2 = 0$ on Γ_S we deduce that $(Dv)|_{\Gamma_S} e_3 \cdot e_3 = 0$. Thus

$$-P_m\Big(\mathbb{T}(v, \pi)|_{\Gamma_S} e_3 \cdot e_3\Big) = \gamma_m \pi,$$

where γ_m is the following modified trace operator:

$$\gamma_m f := P_m(f|_{\Gamma_S}) = f(\cdot, 0) - \frac{1}{|\mathcal{S}|} \int_{\mathcal{S}} f(s', 0)\, ds' \quad (f \in W^{r,q}(\mathcal{F}) \text{ with } r > 1/q). \tag{4.3}$$

This cancelation plays no role in our result and is only used to simplify the calculation.

4.1 The fluid operator

Here we recall some results on the Stokes operator in the L^q framework. Let us introduce the Banach space

$$W^q_{\operatorname{div}}(\mathcal{F}) = \{\varphi \in L^q(\mathcal{F}) \ ; \ \operatorname{div} \varphi \in L^q(\mathcal{F})\},$$

equipped with the norm

$$\|\varphi\|_{W^q_{\operatorname{div}}(\mathcal{F})} := \|\varphi\|_{L^q(\mathcal{F})} + \|\operatorname{div} \varphi\|_{L^q(\mathcal{F})}.$$

We recall (see, for instance, [17, Lemma 1]) that the normal trace can be extended as a continuous and surjective map

$$\begin{aligned} \gamma_n : W^q_{\operatorname{div}}(\mathcal{F}) &\to W^{-1/q,q}(\partial\mathcal{F}), \\ \varphi &\mapsto \varphi \cdot n. \end{aligned}$$

In particular, we can define

$$L^q_\sigma(\mathcal{F}) = \{\varphi \in L^q(\mathcal{F}) \ ; \ \operatorname{div} \varphi = 0 \quad \text{in } \mathcal{F}, \ \varphi \cdot n = 0 \text{ on } \partial\mathcal{F}\}.$$

We have the following Helmholtz-Weyl decomposition (see, for instance Section 3 and Theorem 2 of [17]):

$$L^q(\mathcal{F}) = L^q_\sigma(\mathcal{F}) \oplus G^q(\mathcal{F}), \quad \text{where} \quad G^q(\mathcal{F}) = \left\{\nabla\varphi \ ; \ \varphi \in W^{1,q}(\mathcal{F})\right\}.$$

The corresponding projection operator $\mathcal{P}$ from $L^q(\mathcal{F})$ onto $L^q_\sigma(\mathcal{F})$ can be obtained as

$$\mathcal{P}f = f - \nabla\varphi, \tag{4.4}$$

where $\varphi \in W^{1,q}(\mathcal{F})$ is a solution of the following Neumann problem

$$\Delta\varphi = \operatorname{div} f \quad \text{in } \mathcal{F}, \quad \frac{\partial\varphi}{\partial n} = f \cdot n \quad \text{on } \partial\mathcal{F}, \tag{4.5}$$

that is a solution of

$$\int_{\mathcal{F}} \nabla\varphi \cdot \nabla\psi \ dy = \int_{\mathcal{F}} f \cdot \nabla\psi \ dy \quad (\psi \in W^{1,q'}(\mathcal{F})),$$

where q' is the conjugate exponent of q.

Let us denote by $A_F = \mathcal{P}\Delta$, the Stokes operator in $L^q_\sigma(\mathcal{F})$ with domain

$$\mathcal{D}(A_F) = W^{2,q}(\mathcal{F}) \cap W^{1,q}_0(\mathcal{F}) \cap L^q_\sigma(\mathcal{F}).$$

Theorem 4.1. *Assume $1 < q < \infty$. Then the Stokes operator A_F generates a C^0-semigroup of negative type. Moreover A_F is an $\mathcal{R}$-sectorial operator in $L^q_\sigma(\mathcal{F})$ of angle β for any $\beta \in (0, \pi)$.*

For the proof, we refer to Corollary 1.2 and Theorem 1.4 in [19].

4.2 The structure operator

Let us set

$$\mathcal{X}_S = W^{2,q}_{0,m}(\mathcal{S}) \times L^q_m(\mathcal{S})$$

and let us consider the operator $A_S : \mathcal{D}(A_S) \to \mathcal{X}_S$ defined by

$$\mathcal{D}(A_S) = \left(W^{4,q}(\mathcal{S}) \cap W^{2,q}_{0,m}(\mathcal{S})\right) \times W^{2,q}_{0,m}(\mathcal{S}), \quad A_S = \begin{pmatrix} 0 & \mathrm{Id} \\ -P_m\Delta^2 & \Delta \end{pmatrix},$$

where P_m is defined by (1.8).

Theorem 4.2. *Let us assume that $1 < q < \infty$. Then there exists $\gamma_1 > 0$ such that $A_S - \gamma_1$ is an $\mathcal{R}$-sectorial operator on $\mathcal{X}_S$ of angle $\beta_1 > \pi/2$.*

Proof. We first consider

$$\mathcal{X}^0_S := W^{2,q}_0(\mathcal{S}) \times L^q(\mathcal{S})$$

and the operator A^0_S defined by

$$\mathcal{D}(A^0_S) = \left(W^{4,q}(\mathcal{S}) \cap W^{2,q}_0(\mathcal{S})\right) \times W^{2,q}_0(\mathcal{S}), \quad A^0_S = \begin{pmatrix} 0 & \mathrm{Id} \\ -\Delta^2 & \Delta \end{pmatrix}.$$

Applying Theorem 5.1 in [12], we have that A^0_S is $\mathcal{R}$-sectorial in $\mathcal{X}^0_S$ of angle $\beta_0 > \pi/2$.

Now we can extend A_S on $\mathcal{D}(A^0_S)$ by $\widetilde{A}_S = A^0_S + B_S$ where

$$B_S = \begin{pmatrix} 0 & 0 \\ (\mathrm{Id} - P_m)\Delta^2 & 0 \end{pmatrix}, \quad (\mathrm{Id} - P_m)\Delta^2\eta_1 = \frac{1}{|\mathcal{S}|}\int_{\partial\mathcal{S}} (\nabla\Delta\eta_1) \cdot n_{\mathcal{S}} \ ds.$$

Using standard result on the trace operator, we see that B_S satisfies the hypotheses of Proposition 3.5 and in particular for any $a > 0$ there exists $b > 0$ such that (3.4) holds. Therefore, there exists $\gamma_1 > 0$ such that $\widetilde{A}_S - \gamma_1$ is an $\mathcal{R}$-sectorial operator on $\mathcal{X}_S^0$ of angle β_0.

Let $\lambda \neq 0$, $(g_1, g_2) \in \mathcal{X}_S$ and $(\eta_1, \eta_2) \in \mathcal{D}(A_S^0)$ such that

$$(\lambda - \widetilde{A}_S) \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}.$$

We can write this equation as

$$\begin{aligned}
&\lambda\eta_1 - \eta_2 = g_1 \qquad \text{in } \mathcal{S}, \\
&\lambda\eta_2 + P_m \Delta^2 \eta_1 - \Delta\eta_2 = g_2, \quad \text{in } \mathcal{S}, \\
&\eta_1 = \nabla_s \eta_1 \cdot n_{\mathcal{S}} = \eta_2 = \nabla_s \eta_2 \cdot n_{\mathcal{S}} = 0 \qquad \text{on } \partial\mathcal{S}.
\end{aligned}$$

Integrating the first two equations over $\mathcal{S}$ we find that $(\eta_1, \eta_2) \in \mathcal{D}(A_S)$. Thus

$$\left[(\lambda - \widetilde{A}_S)^{-1} \right]_{|\mathcal{X}_S} = (\lambda - A_S)^{-1}.$$

Using basic properties on $\mathcal{R}$-boundedness, we deduce the result. $\square$

4.3 The fluid-structure operator

In this subsection we rewrite (4.2) in a suitable operator form. The idea is to eliminate the pressure from both the fluid and the structure equations. To eliminate the pressure from the fluid equation we use the Leray projector $\mathcal{P}$ defined in equation (4.4). Following [39], we first decompose the fluid velocity into two parts $\mathcal{P}v$ and $(\mathrm{Id} - \mathcal{P})v$. Next, we split the pressure into two parts, one which depends on $\mathcal{P}v$ and another part which depends on η_2. This will lead us to an equation of evolution for $(\mathcal{P}v, \eta_1, \eta_2)$ and an algebraic equation for $(\mathrm{Id} - \mathcal{P})v$.

The advantage of this formulation is that the $\mathcal{R}$-boundedness of the fluid-structure operator can be obtained just by using the fact that the operators A_F and A_S are $\mathcal{R}$-sectorial and a perturbation argument. This idea has been used in several fluid-solid interaction problems in the Hilbert space setting as well as in L^q-setting (see, for instance, [41, 27, 36, 34] and the references therein).

Let us consider the following problem :

$$\left\{ \begin{aligned}
-\operatorname{div} \mathbb{T}(w, \psi) &= f \quad \text{in } \mathcal{F}, \\
\operatorname{div} w &= 0 \quad \text{in } \mathcal{F}, \\
w &= \mathcal{T} g \quad \text{on } \partial\mathcal{F}, \\
\int_{\mathcal{F}} \psi \, dx &= 0.
\end{aligned} \right. \tag{4.6}$$

From [43, Proposition 2.3, p. 35], we have the following result:

Lemma 4.3. *Assume $1 < q < \infty$. For any $f \in L^q(\mathcal{F})$ and $g \in W^{2,q}_{0,m}(\mathcal{S})$, the system (4.6) admits a unique solution $(w, \psi) \in W^{2,q}(\mathcal{F}) \times W^{1,q}_m(\mathcal{F})$.*

This allows us to introduce the following operators: we consider

$$D_{\mathrm{v}} \in \mathcal{L}(W^{2,q}_{0,m}(\mathcal{S}), W^{2,q}(\mathcal{F})) \quad \text{and} \quad D_{\mathrm{p}} \in \mathcal{L}(W^{2,q}_{0,m}(\mathcal{S}), W^{1,q}_m(\mathcal{F})) \tag{4.7}$$

defined by

$$D_{\mathrm{v}} g = w, \quad D_{\mathrm{p}} g = \psi, \tag{4.8}$$

where (w, ψ) is the solution to the problem (4.6) associated with g and in the case $f = 0$.

Second, we consider the Neumann problem

$$\Delta\varphi = 0 \text{ in } \mathcal{F}, \quad \frac{\partial\varphi}{\partial n} = h \text{ on } \partial\mathcal{F}, \quad \int_{\mathcal{F}} \varphi \, dx = 0. \tag{4.9}$$

Let us denote by N the operator defined by

$$Nh = \varphi. \tag{4.10}$$

Using classical results (see for instance Theorem 4.2 and Theorem 4.3 of [33]), we have the following properties of N:

$$\begin{gathered} N \in \mathcal{L}(W_m^{1-1/q,q}(\partial\mathcal{F}), W_m^{2,q}(\mathcal{F})), \quad N \in \mathcal{L}(W_m^{-1/q,q}(\partial\mathcal{F}), W_m^{1,q}(\mathcal{F})), \\ N \in \mathcal{L}(L_m^q(\partial\mathcal{F}), W_m^{1+1/q-\varepsilon,q}(\mathcal{F})), \end{gathered} \tag{4.11}$$

for any $\varepsilon > 0$. We recall that $W_m^{-1/q,q}(\partial\mathcal{F})$ is defined as follows:

$$W_m^{-1/q,q}(\partial\mathcal{F}) = \left\{ h \in W^{-1/q,q}(\partial\mathcal{F}) \ ; \ \langle h, 1 \rangle_{W^{-1/q,q}, W^{1-1/q',q'}} = 0 \right\}, \tag{4.12}$$

where q' the conjugate exponent of q.

We also define

$$N_S g = Nh \quad \text{with} \quad h(y) = \begin{cases} g(s) & \text{if} \quad y = (s,0) \in \Gamma_S, \\ 0 & \text{if} \quad y \in \Gamma_0. \end{cases} \tag{4.13}$$

From the above properties of N, we deduce that

$$N_S \in \mathcal{L}(L_m^q(\mathcal{S}), W_m^{1+1/q-\varepsilon,q}(\mathcal{F})), \tag{4.14}$$

for any $\varepsilon > 0$.

Finally, we introduce the operator $N_{HW} \in \mathcal{L}(L^q(\mathcal{F}), W_m^{1,q}(\mathcal{F}))$ defined by

$$N_{HW} f = \varphi, \tag{4.15}$$

where φ solves (4.5).

Using the above operators, we can obtain the following proposition. The proof is similar to the proof of [36, Proposition 3.7]. For the sake of completeness, we provide a short proof here.

Proposition 4.4. *Let $1 < p, q < \infty$. Assume*

$$\begin{gathered} v \in W_{p,q}^{1,2}((0,\infty);\mathcal{F}), \quad \pi \in L^p(0,\infty; W_m^{1,q}(\mathcal{F})), \\ \eta_1 \in W_{p,q}^{2,4}((0,\infty);\mathcal{S}), \quad \eta_2 \in W_{p,q}^{1,2}((0,\infty);\mathcal{S}). \end{gathered}$$

Then (v, π, η_1, η_2) is a solution of (4.2) if and only if

$$\begin{cases} \mathcal{P}v' = A_F \mathcal{P}v - A_F \mathcal{P} D_{\mathrm{v}} \eta_2 + \mathcal{P}f & \text{in } (0,\infty), \\ \partial_t \eta_1 = \eta_2 & \text{in } (0,\infty), \\ (\mathrm{Id} + \gamma_m N_S)\partial_t \eta_2 + P_m \Delta^2 \eta_1 - \Delta \eta_2 = \gamma_m N(\nu \Delta \mathcal{P}v \cdot n) + P_m h + \gamma_m N_{HW} f & \text{in } (0,\infty), \\ [\mathcal{P}v, \eta_1, \eta_2]^\top (0,\cdot) = [\mathcal{P}v^0, \eta_1^0, \eta_2^0]^\top \\ (\mathrm{Id} - \mathcal{P})v = (\mathrm{Id} - \mathcal{P}) D_{\mathrm{v}} \eta_2 & \text{in } (0,\infty), \\ \pi = N(\nu \Delta \mathcal{P}v \cdot n) - N_S \partial_t \eta_2 + N_{HW} f & \text{in } (0,\infty). \end{cases} \tag{4.16}$$

Proof. Considering the equation satisfied by $(v - D_{\mathrm{v}} g, \pi - D_{\mathrm{p}} g)$, we obtain $(4.16)_1$ and $(4.16)_5$. Using (4.4) and (4.5), it follows that $\Delta(\mathrm{Id} - \mathcal{P})v = 0$ in $\mathcal{F}$. Thus applying the divergence and normal trace operators to (4.6), we infer that

$$\Delta \psi = \operatorname{div} f \quad \text{in } \mathcal{F}, \qquad \frac{\partial \psi}{\partial n} = f \cdot n + \nu \Delta \mathcal{P}v \cdot n - \mathcal{T} \partial_t \eta_2 \cdot n \text{ on } \partial\mathcal{F}. \tag{4.17}$$

Note that $\operatorname{div} \Delta \mathcal{P}v = 0$ and therefore $\Delta \mathcal{P}v \cdot n$ belongs to $W_m^{-1/q,q}(\partial\mathcal{F})$. The expression of ψ then follows from the definition of the operators N, N_S and N_{HW} defined in (4.10), (4.13) and (4.15) respectively. Finally, using the expression of the pressure π we can rewrite the equation satisfied by η_2 as in $(4.16)_3$. ☐

In the literature, the operator

$$M_S := \operatorname{Id} + \gamma_m N_S$$

is known as the added mass operator. We are going to show that it is invertible.

Lemma 4.5. *The operator $M_S = \operatorname{Id} + \gamma_m N_S \in \mathcal{L}(L^q_m(\mathcal{S}))$ is an automorphism in $W^{s,q}_m(\mathcal{S})$ for any $s \in [0,1)$. Moreover, $M_S^{-1} - \operatorname{Id} \in \mathcal{L}(L^q_m(\mathcal{S}), W^{s,q}_m(\mathcal{S}))$, for any $s \in [0,1)$. In particular, $M_S^{-1} - \operatorname{Id}$ is a compact operator on $L^q_m(\mathcal{S})$.*

Proof. At first, we show that M_S is an invertible operator on $L^q_m(\mathcal{S})$. Since

$$\gamma_m N_S \in \mathcal{L}(L^q_m(\mathcal{S}), W^{1-\varepsilon,q}_m(\mathcal{S})),$$

for any $\varepsilon \in (0,1]$, it is sufficient to show that the kernel of M_S is reduced to $\{0\}$: assume

$$(\operatorname{Id} + \gamma_m N_S) f = 0. \tag{4.18}$$

Then $f \in W^{1-\varepsilon,q}_m(\mathcal{S}) \subset L^2_m(\mathcal{S})$ for ε small enough. In particular (see (4.13)), $\vartheta = N_S f \in H^1(\mathcal{F})$ is the weak solution of

$$\Delta \vartheta = 0 \text{ in } \mathcal{F}, \quad \frac{\partial \vartheta}{\partial n} = f \text{ on } \Gamma_S, \quad \frac{\partial \vartheta}{\partial n} = 0 \text{ on } \Gamma_0.$$

Multiplying (4.18) by f and using the above system, we deduce after integration by parts,

$$\int_{\mathcal{S}} [(\operatorname{Id} + \gamma_m N_S) f]\, f \; ds = \int_{\mathcal{S}} f^2 \; ds + \int_{\mathcal{F}} |\nabla \vartheta|^2 \; dy = 0.$$

Thus $f = 0$ and M_S is an invertible operator on $L^q_m(\mathcal{S})$. Let $s \in [0,1)$ and $f_0 \in W^{s,q}_m(\mathcal{S})$. By the above argument, there exists a unique $f \in L^q_m(\mathcal{S})$ such that

$$(\operatorname{Id} + \gamma_m N_S) f = f_0.$$

As $\gamma_m N_S f \in W^{s,q}_m(\mathcal{S})$ we conclude that $f \in W^{s,q}_m(\mathcal{S})$. Thus M_S is an invertible operator on $W^{s,q}_m(\mathcal{S})$. Finally, the compactness of the operator $M_S^{-1} - \operatorname{Id}$ follows from the following identity

$$M_S^{-1} - \operatorname{Id} = M_S^{-1} - M_S^{-1} M_S = -M_S^{-1} \gamma_m N_S.$$

□

We are now in a position to rewrite the system (4.2) in a suitable operator form. Let us set

$$\mathcal{X} = L^q_\sigma(\mathcal{F}) \times \mathcal{X}_S \tag{4.19}$$

and consider the operator $\mathcal{A}_{FS} : \mathcal{D}(\mathcal{A}_{FS}) \to \mathcal{X}$ defined by

$$\mathcal{D}(\mathcal{A}_{FS}) = \Big\{ [v, \eta_1, \eta_2]^\top \in \big[W^{2,q}(\mathcal{F}) \cap L^q_\sigma(\mathcal{F}) \big] \times \mathcal{D}(A_S) \; ; \; v - \mathcal{P} D_{\mathrm{v}} \eta_2 \in \mathcal{D}(A_F) \Big\},$$

and

$$\mathcal{A}_{FS} = \mathcal{A}^0_{FS} + \mathcal{B}_{FS},$$

with

$$\mathcal{A}^0_{FS} := \begin{bmatrix} A_F & 0 & -A_F \mathcal{P} D_{\mathrm{v}} \\ 0 & 0 & \operatorname{Id} \\ 0 & -P_m \Delta^2 & \Delta \end{bmatrix} \tag{4.20}$$

and

$$\mathcal{B}_{FS} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ M_S^{-1} \gamma_m N(\nu \Delta(\cdot) \cdot n) & -(M_S^{-1} - \operatorname{Id}) P_m \Delta^2 & (M_S^{-1} - \operatorname{Id}) \Delta \end{bmatrix}. \tag{4.21}$$

Combining Proposition 4.4 and Lemma 4.5, we can rewrite the system (4.2) as

$$\frac{d}{dt}\begin{bmatrix}\mathcal{P}v\\ \eta_1\\ \eta_2\end{bmatrix} = \mathcal{A}_{FS}\begin{bmatrix}\mathcal{P}v\\ \eta_1\\ \eta_2\end{bmatrix} + \begin{bmatrix}\mathcal{P}f\\ 0\\ \overline{h}\end{bmatrix}, \quad \begin{bmatrix}\mathcal{P}v\\ \eta_1\\ \eta_2\end{bmatrix}(0) = \begin{bmatrix}\mathcal{P}v^0\\ \eta_1^0\\ \eta_2^0\end{bmatrix}, \tag{4.22}$$

$$(\mathrm{Id}-\mathcal{P})v = (\mathrm{Id}-\mathcal{P})D_{\mathrm{v}}\eta_2, \tag{4.23}$$

$$\pi = N(\nu\Delta\mathcal{P}v\cdot n) - N_S\partial_t\eta_2 + N_{HW}f, \tag{4.24}$$

where

$$\overline{h} = M_S^{-1}P_m h + M_S^{-1}\gamma_m N_{HW}f. \tag{4.25}$$

4.4 $\mathcal{R}$-sectoriality of the operator $\mathcal{A}_{FS}$.

In this subsection we prove the following theorem

Theorem 4.6. *Let $1 < q < \infty$. There exists $\gamma_2 > 0$ such that $\mathcal{A}_{FS} - \gamma_2$ is an $\mathcal{R}$-sectorial operator in $\mathcal{X}$ of angle $> \pi/2$. Moreover the operator $\mathcal{A}_{FS}$ generates an exponentially stable semigroup on $\mathcal{X}$: there exist constants $C > 0$ and $\beta_0 > 0$ such that*

$$\left\|e^{t\mathcal{A}_{FS}}(v^0,\eta_1^0,\eta_2^0)^\top\right\|_{\mathcal{X}} \leqslant Ce^{-\beta_0 t}\left\|(v^0,\eta_1^0,\eta_2^0)^\top\right\|_{\mathcal{X}} \quad (t\geqslant 0). \tag{4.26}$$

Proof. Observe that

$$\lambda\left(\lambda-\mathcal{A}_{FS}^0\right)^{-1} = \begin{bmatrix}\lambda(\lambda-A_F)^{-1} & -A_F(\lambda-A_F)^{-1}\mathcal{P}\widetilde{D}_{\mathrm{v}}\lambda(\lambda-A_S)^{-1}\\ 0 & \lambda(\lambda-A_S)^{-1}\end{bmatrix},$$

where $\widetilde{D}_{\mathrm{v}}\left[f_1,f_2\right]^\top = D_{\mathrm{v}}f_2$. Using a standard transposition method and Lemma 4.3, we see that

$$D_{\mathrm{v}} \in \mathcal{L}(L^q_m(\mathcal{S}), L^q(\mathcal{F})). \tag{4.27}$$

Therefore by Theorem 4.1 and Theorem 4.2, there exists $\gamma > 0$ such that $\mathcal{A}^0_{FS} - \gamma$ is $\mathcal{R}$-sectorial operator in $\mathcal{X}$ of angle $> \pi/2$.

Next, we want to show $\mathcal{B}_{FS} \in \mathcal{L}(\mathcal{D}(\mathcal{A}_{FS}),\mathcal{X})$ is a compact operator. Assume $[v,\eta_1,\eta_2]^\top \in \mathcal{D}(\mathcal{A}_{FS})$. Then $\Delta v \in L^q(\mathcal{F})$ and $\operatorname{div}\Delta v = 0$ and thus from the trace result recalled in Section 4.1,

$$(\Delta v)\cdot n \in W_m^{-1/q,q}(\partial\mathcal{F}).$$

This yields $N((\Delta v)\cdot n) \in W^{1,q}_m(\mathcal{F})$, $\gamma_m N((\Delta v)\cdot n) \in W_m^{1-1/q,q}(\mathcal{S})$ and, using Lemma 4.5,

$$M_S^{-1}\gamma_m N((\Delta v)\cdot n) \in W_m^{1-1/q,q}(\mathcal{S}).$$

On the other hand, using again Lemma 4.5, we deduce

$$(M_S^{-1}-\mathrm{Id})P_m\Delta^2 \in \mathcal{L}(W^{4,q}(\mathcal{S}), W_m^{1-\varepsilon,q}(\mathcal{S})), \quad (M_S^{-1}-\mathrm{Id})\Delta \in \mathcal{L}(W_m^{2,q}(\mathcal{S}), W_m^{1-\varepsilon,q}(\mathcal{S}))$$

for any $\varepsilon > 0$. Therefore, $\mathcal{B}_{FS} \in \mathcal{L}(\mathcal{D}(\mathcal{A}_{FS}),\mathcal{X})$ is a compact operator and by [14, Chapter III, Lemma 2.16], $\mathcal{B}_{FS}$ is a $\mathcal{A}^0_{FS}$-bounded operator with relative bound 0. Finally, using Proposition 3.5 we conclude the first part of the theorem. In particular $\mathcal{A}_{FS}$ generates an analytic semigroup and to show the second part of the theorem, it is sufficient to show that

$$\mathbb{C}^+ = \{\lambda\in\mathbb{C}\ ;\ \mathrm{Re}\lambda \geqslant 0\} \subset \rho(\mathcal{A}_{FS}).$$

Moreover, using that $\mathcal{A}_{FS}$ has a compact resolvent and the Fredholm alternative theorem, we can show the above relation by proving that $\ker(\lambda-\mathcal{A}_{FS}) = \{0\}$ for $\lambda\in\mathbb{C}^+$. Assume $\lambda\in\mathbb{C}^+$ and

$$(v,\pi,\eta_1,\eta_2) \in W^{2,q}(\mathcal{F})\times W^{1,q}_m(\mathcal{F})\times W^{4,q}_m(\mathcal{S})\times W^{2,q}_m(\mathcal{S})$$

satisfy

$$\begin{cases} \lambda v - \operatorname{div} \mathbb{T}(v,\pi) = 0 & \text{in } \mathcal{F}, \\ \operatorname{div} v = 0 & \text{in } \mathcal{F}, \\ v = \mathcal{T}\eta_2 & \text{on } \partial\mathcal{F}, \\ \lambda\eta_1 - \eta_2 = 0 & \text{in } \mathcal{S}, \\ \lambda\eta_2 + P_m\Delta^2\eta_1 - \Delta\eta_2 = \gamma_m\pi & \text{in } \mathcal{S}, \\ \eta_1 = \nabla_s\eta_1 \cdot n_{\mathcal{S}} = 0 & \text{on } \partial\mathcal{S}. \end{cases} \tag{4.28}$$

First we notice that

$$(v,\pi,\eta_1,\eta_2) \in W^{2,2}(\mathcal{F}) \times W^{1,2}_m(\mathcal{F}) \times W^{4,2}_m(\mathcal{S}) \times W^{2,2}_m(\mathcal{S}). \tag{4.29}$$

If $q \geqslant 2$ then it is a consequence of Hölder's inequality. Let us assume that $1 < q < 2$ and let us take $\lambda_0 \in \rho(\mathcal{A}_{FS})$ (see Theorem 4.6). We have

$$(\lambda_0 - \mathcal{A}_{FS})[v,\eta_1,\eta_2]^\top = (\lambda_0 - \lambda)[v,\eta_1,\eta_2]^\top$$

By following the calculation done in Section 4.3, we see that the system (4.28) can be written as

$$\begin{cases} (\lambda_0 - \mathcal{A}_{FS}) \begin{bmatrix} \mathcal{P}v \\ \eta_1 \\ \eta_2 \end{bmatrix} = (\lambda_0 - \lambda) \begin{bmatrix} \mathcal{P}v \\ \eta_1 \\ \eta_2 \end{bmatrix}, \\ (\operatorname{Id} - \mathcal{P})v = (\operatorname{Id} - \mathcal{P})D_{\mathrm{v}}\eta_2, \\ \pi = N(\nu\Delta\mathcal{P}v \cdot n) - \lambda N_{\mathcal{S}}\eta_2. \end{cases}$$

Since $W^{2,q}(\mathcal{F}) \subset L^2(\mathcal{F})$, $W^{2,q}(\mathcal{S}) \subset L^2(\mathcal{S})$ and $(\lambda_0 - \mathcal{A}_{FS})$ is invertible, we deduce (4.29).

Using (4.29), we can multiply $(4.28)_1$ by $\overline{v}$ and $(4.28)_5$ by $\overline{\eta_2}$, and we obtain after integration by parts:

$$\lambda \int_{\mathcal{F}} |v|^2 \, dy + 2\nu \int_{\mathcal{F}} |D(v)|^2 \, dy + \lambda \int_{\mathcal{S}} |\eta_2|^2 \, ds + \overline{\lambda} \int_{\mathcal{S}} |\Delta_s\eta_1|^2 \, ds + \int_{\mathcal{S}} |\nabla_s\eta_2|^2 \, ds = 0.$$

Since $\mathrm{Re}\lambda \geqslant 0$, from the above equality and using the boundary conditions we obtain that $v = \pi = \eta_1 = \eta_2 = 0$. This completes the proof of the theorem. □

In order to obtain a result of well-posedness on the system (4.2), we need to impose some compatibility conditions on the data:

$$\eta_1^0 = \nabla_s\eta_1^0 \cdot n_{\mathcal{S}} = 0 \quad \text{on } \partial\mathcal{S}, \qquad \int_{\mathcal{S}} \eta_1^0 \, ds = 0, \qquad \int_{\mathcal{S}} \eta_2^0 \, ds = 0, \qquad \operatorname{div} v^0 = 0 \text{ in } \mathcal{F}, \tag{4.30}$$

and

$$\begin{cases} v^0 \cdot n = \mathcal{T}\eta_2 \cdot n \quad \text{on } \partial\mathcal{F} & \text{if } \dfrac{1}{p} + \dfrac{1}{2q} > 1, \\ v^0 = \mathcal{T}\eta_2 \quad \text{on } \partial\mathcal{F}, \quad \eta_2^0 = 0 \quad \text{on } \partial\mathcal{S} & \text{if } \dfrac{1}{p} + \dfrac{1}{2q} < 1, \\ \nabla_s\eta_2^0 \cdot n_{\mathcal{S}} = 0 \text{ on } \partial\mathcal{S} & \text{if } \dfrac{1}{p} + \dfrac{1}{2q} < \dfrac{1}{2}. \end{cases} \tag{4.31}$$

We deduce from Theorem 4.6 the following result

Corollary 4.7. *Let $p,q \in (1,\infty)$ with $\frac{1}{p} + \frac{1}{2q} \neq \frac{1}{2}$, $\frac{1}{p} + \frac{1}{2q} \neq 1$ and let $\beta \in [0,\beta_0]$, where β_0 is the constant in Theorem 4.6. Assume*

$$v^0 \in B^{2(1-1/p)}_{q,p}(\mathcal{F}), \quad \eta_1^0 \in B^{2(2-1/p)}_{q,p}(\mathcal{S}), \quad \eta_2^0 \in B^{2(1-1/p)}_{q,p}(\mathcal{S}), \tag{4.32}$$

$$f \in L^p_\beta(0,\infty;L^q(\mathcal{F})), \quad h \in L^p_\beta(0,\infty;L^q(\mathcal{S}))$$

satisfy the compatibility conditions (4.30) *and* (4.31)*. Then the system* (4.2) *admits a unique strong solution*

$$v \in W^{1,2}_{p,q,\beta}((0,\infty);\mathcal{F}), \quad \pi \in L^p_\beta(0,\infty;W^{1,q}_m(\mathcal{F})),$$
$$\eta_1 \in W^{2,4}_{p,q,\beta}((0,\infty);\mathcal{S}) \cap L^p(0,\infty;L^q_m(\mathcal{S})),$$
$$\eta_2 \in W^{1,2}_{p,q,\beta}((0,\infty);\mathcal{S}) \cap L^p(0,\infty;L^q_m(\mathcal{S})).$$

Moreover, there exists a constant C_L depending on p,q and the geometry such that

$$\begin{aligned}\|v\|_{W^{1,2}_{p,q,\beta}((0,\infty);\mathcal{F})} + \|\pi\|_{L^p_\beta(0,\infty;W^{1,q}_m(\mathcal{F}))} + \|\eta_1\|_{W^{2,4}_{p,q,\beta}((0,\infty);\mathcal{S})} + \|\eta_2\|_{W^{1,2}_{p,q,\beta}((0,\infty);\mathcal{S})} \\ \leqslant C_L\Big(\|v^0\|_{B^{2(1-1/p)}_{q,p}(\mathcal{F})} + \|\eta_1^0\|_{B^{2(2-1/p)}_{q,p}(\mathcal{S})} + \|\eta_2^0\|_{B^{2(1-1/p)}_{q,p}(\mathcal{S})} \\ + \|f\|_{L^p_\beta(0,\infty;L^q(\mathcal{F}))} + \|h\|_{L^p_\beta(0,\infty;L^q(\mathcal{S}))}\Big). \quad (4.33)\end{aligned}$$

Proof. Let us first consider the case $\beta = 0$. Using (4.25), (4.15) and Lemma 4.5 we can also verify that $\overline{h} \in L^p(0,\infty;L^q_m(\mathcal{S}))$.

The compatibility conditions (4.30), (4.31) and the interpolation results [3, Theorem 3.4] and [4, Theorem 4.9.1 and Example 4.9.3]) yield

$$\left[\mathcal{P}v^0, \eta_1^0, \eta_2^0\right]^\top \in (\mathcal{X}, \mathcal{D}(\mathcal{A}_{FS}))_{1-1/p,p}$$

and

$$\left[\mathcal{P}f, 0, \overline{h}\right]^\top \in L^p(0,\infty;\mathcal{X}).$$

From Theorem 4.6, we know that $\mathcal{A}_{FS}$ generates an analytic exponentially stable semigroup on $\mathcal{X}$ and is a $\mathcal{R}$-sectorial operator on $\mathcal{X}$. Therefore by Corollary 3.4

$$(\mathcal{P}v, \eta_1, \eta_2) \in L^p(0,\infty;\mathcal{D}(\mathcal{A}_{FS})) \cap W^{1,p}(0,\infty;\mathcal{X}).$$

We deduce from (4.23), (4.7) and (4.27) that $v \in W^{1,2}_{p,q}((0,\infty);\mathcal{F})$ and next using relations (4.11), (4.14) and (4.15), we obtain $\pi \in L^p(0,\infty;W^{1,q}_m(\mathcal{F}))$.

The case $\beta > 0$ can be reduced to the previous case by multiplying all the functions by $e^{\beta t}$ and using the fact that $\mathcal{A}_{FS} + \beta$ is a $\mathcal{R}$-sectorial operator and generates an exponentially stable semigroup. $\square$

5 Local in time existence

The aim of this section is to prove Theorem 1.1 and Theorem 2.2. Throughout this section we assume the following

Assumption 5.1. $\eta_1^0 = 0$, $(p,q) \in (1,\infty)$ *satisfies* (1.15) *and* (η_2^0, v^0) *satisfies* (2.18), (2.19), (2.20).

For $T > 0$ and $R > 0$, we define $\mathbb{S}_{T,R}$ as follows

$$\mathbb{S}_{T,R} := \Big\{(f,h) \in L^p(0,T;L^q(\mathcal{F})) \times L^p(0,T;L^q(\mathcal{S}))\ ;\ \|f\|_{L^p(0,T;L^q(\mathcal{F}))} + \|h\|_{L^p(0,T;L^q(\mathcal{S}))} \leqslant R\Big\}. \quad (5.1)$$

In order to prove Theorem 2.2, we show that for R fixed and for T small, we can define the map

$$\mathcal{N}_{T,R} : \mathbb{S}_{T,R} \longrightarrow \mathbb{S}_{T,R} \quad (f,h) \longmapsto (F(v,\pi,\eta), H(v,\pi,\eta)), \quad (5.2)$$

where (v,π,η) is the solution to the system (4.2) in $(0,T)\times\mathcal{F}$ (see Corollary 4.7) and where F and H are given by (2.13)-(2.14). Then we show that for T small enough and R fixed $\mathcal{N}_{T,R}(\mathbb{S}_{T,R}) \subset \mathbb{S}_{T,R}$ (see Proposition 5.2 below) and that, $\mathcal{N}_{T,R}|_{\mathbb{S}_{T,R}}$ is a strict contraction (see Proposition 5.3 below). This shows that $\mathcal{N}_{T,R}$ admits a unique fixed point and allows us to deduce Theorem 2.2.

First, we deduce from Corollary 4.7 that

$$\|v\|_{W^{1,2}_{p,q}((0,T);\mathcal{F})} + \|\pi\|_{L^p(0,T;W^{1,q}_m(\mathcal{F}))} + \|\eta\|_{W^{2,4}_{p,q}((0,T);\mathcal{S})} \leqslant C(\|v^0\|_{B^{2(1-1/p)}_{q,p}(\mathcal{F})} + \|\eta_2^0\|_{B^{2(1-1/p)}_{q,p}(\mathcal{S})} + R). \tag{5.3}$$

We take in what follows

$$R := \|v^0\|_{B^{2(1-1/p)}_{q,p}(\mathcal{F})} + \|\eta_2^0\|_{B^{2(1-1/p)}_{q,p}(\mathcal{S})}$$

and the constants below may depend on R, but not on T. In order to simplify the computation, we also assume that $T \in (0,1)$.

With these conventions, by using [45, (7), p.196], we have that for any $s_1 \in (0, 2(1-1/p))$, with s_1 not an integer,

$$\|\eta\|_{L^\infty(0,T;W^{2+s_1,q}(\mathcal{S}))} + \|\eta\|_{W^{1,\infty}(0,T;W^{s_1,q}(\mathcal{S}))} + \|v\|_{L^\infty(0,T;W^{s_1,q}(\mathcal{F}))} \leqslant C. \tag{5.4}$$

Since $\eta(0,\cdot) = 0$, we have

$$\|\eta\|_{L^\infty(0,T;W^{2,q}(\mathcal{S}))} \leqslant CT^{1/p'} \|\partial_t \eta\|_{L^p(0,T;W^{2,q}(\mathcal{S}))} \leqslant CT^{1/p'}. \tag{5.5}$$

Thus, by interpolation between (5.4) and (5.5) ([44, Theorem 2, p. 317]), we deduce that for any $s_1 \in (0, 2(1-1/p))$, there exists $\varepsilon = \varepsilon(s_1) > 0$ such that

$$\|\eta\|_{L^\infty(0,T;W^{2+s_1,q}(\mathcal{S}))} \leqslant CT^\varepsilon. \tag{5.6}$$

From (1.15), there exists $s_1 \in (0, 2(1-1/p))$, such that $s_1 + 1 > 3/q$ and thus with the Sobolev embeddings, we deduce that

$$\|\eta\|_{L^\infty(0,T;C^1(\overline{\mathcal{S}}))} \leqslant CT^\varepsilon. \tag{5.7}$$

Therefore, for T small enough, $\eta(t,\cdot)$ satisfies (2.1) for all $t \in [0,T]$ where c_0 is defined in (2.5). We can thus construct X by (2.6) so that $X(t,\cdot)$ is a C^1-diffeomorphism from $\mathcal{F}$ onto $\mathcal{F}(\eta(t))$. We can also consider $F(v,\pi,\eta)$ and $H(v,\pi,\eta))$ given by (2.13)-(2.14). In order to estimate these expressions, we also note that by (real or complex) interpolation ([44, Theorem 2, p. 317]) for $\theta \in (0,1)$,

$$\|v(t,\cdot)\|_{W^{s_2,q}(\mathcal{F})} \leqslant C\|v(t,\cdot)\|^{1-\theta}_{W^{s_1,q}(\mathcal{F})} \|v(t,\cdot)\|^\theta_{W^{2,q}(\mathcal{F})}, \quad s_2 = 2\theta + (1-\theta)s_1,$$

if s_2 is not an integer. We can find $\theta \in (0,1/3)$ and $s_1 \in (0, 2(1-1/p))$ such that $s_2 \geqslant 2/q$ so that by Sobolev embeddings,

$$\|v\|_{L^{3p}(0,T;L^{3q}(\mathcal{F}))} \leqslant CT^\varepsilon \|v\|^{1-\theta}_{L^\infty(0,T;W^{s_1,q}(\mathcal{F}))} \|v\|^\theta_{L^p(0,T;W^{2,q}(\mathcal{F}))} \leqslant CT^\varepsilon \tag{5.8}$$

and similarly,

$$\|\nabla_s^2 \eta\|_{L^{3p}(0,T;L^{3q}(\mathcal{S}))} + \|\partial_t \eta\|_{L^{3p}(0,T;L^{3q}(\mathcal{S}))} \leqslant CT^\varepsilon, \tag{5.9}$$

$$\|\nabla v\|_{L^{3p/2}(0,T;L^{3q/2}(\mathcal{F}))} + \|\nabla_s^3 \eta\|_{L^{3p/2}(0,T;L^{3q/2}(\mathcal{S}))} + \|\nabla_s \partial_t \eta\|_{L^{3p/2}(0,T;L^{3q/2}(\mathcal{S}))} \leqslant CT^\varepsilon. \tag{5.10}$$

We are now in position to prove the following result:

Proposition 5.2. *With the above assumptions (in particular Assumption 5.1), there exists $T > 0$ small enough such that the map $\mathcal{N}_{T,R}$ (see (5.2)) is well-defined and satisfies $\mathcal{N}_{T,R}(\mathbb{S}_{T,R}) \subset \mathbb{S}_{T,R}$.*

Proof. From (2.4) and (5.7), we deduce that for $T > 0$ small enough

$$\begin{aligned}\|\nabla X - I_3\|_{L^\infty(0,T;C^0(\overline{\mathcal{F}}))} &+ \|\nabla Y(X) - I_3\|_{L^\infty(0,T;C^0(\overline{\mathcal{F}}))} \\ &+ \|a(X) - I_3\|_{L^\infty(0,T;C^0(\overline{\mathcal{F}}))} + \|b - I_3\|_{L^\infty(0,T;C^0(\overline{\mathcal{F}}))} \leqslant CT^\varepsilon,\end{aligned} \tag{5.11}$$

$$\|\det(\nabla X) - 1\|_{L^\infty(0,T;C^0(\overline{\mathcal{F}}))} \leqslant CT^\varepsilon, \quad \frac{1}{2} \leqslant \|\det \nabla X\|_{L^\infty(0,T;C^0(\overline{\mathcal{F}}))} \leqslant \frac{3}{2}, \tag{5.12}$$

$$\|\nabla X\|_{L^\infty(0,T;C^0(\overline{\mathcal{F}}))} + \|\nabla Y(X)\|_{L^\infty(0,T;C^0(\overline{\mathcal{F}}))} + \|a(X)\|_{L^\infty(0,T;C^0(\overline{\mathcal{F}}))} + \|b\|_{L^\infty(0,T;C^0(\overline{\mathcal{F}}))} \leqslant C. \tag{5.13}$$

We recall that a and b are defined by (2.11).

By using standard properties of linear algebra, we have that

$$a(X) = \frac{\nabla X}{\det(\nabla X)} \tag{5.14}$$

and thus for all i, j, k,

$$\left| \frac{\partial a_{ik}}{\partial x_j}(X) \right| \leqslant C \left| \nabla^2 X \right| \leqslant C \left(|\eta| + |\nabla_s \eta| + |\nabla_s^2 \eta| \right), \tag{5.15}$$

$$\left| \frac{\partial^2 a_{ik}}{\partial x_j^2}(X) \right| \leqslant C \left(\left| \nabla^2 X \right|^2 + \left| \nabla^3 X \right| \right) \leqslant C \left(\left(|\eta| + |\nabla_s \eta| + |\nabla_s^2 \eta| \right)^2 + |\nabla_s^3 \eta| \right), \tag{5.16}$$

$$|\partial_t a(X)| \leqslant C \left(\left| \nabla^2 X \right| + |\nabla \partial_t X| \right) \leqslant C \left(|\eta| + |\nabla_s \eta| + |\nabla_s^2 \eta| + |\nabla_s \partial_t \eta| \right). \tag{5.17}$$

We also have

$$|\partial_t Y(X)| \leqslant C \left| \partial_t X \right| \leqslant C |\partial_t \eta|, \tag{5.18}$$

$$\left| \frac{\partial^2 Y_\ell}{\partial x_j^2}(X) \right| \leqslant C \left| \nabla^2 X \right| \leqslant C \left(|\eta| + |\nabla_s \eta| + |\nabla_s^2 \eta| \right). \tag{5.19}$$

Combining the above estimates with (5.8), (5.9) and (5.10), we deduce that F defined by (2.13) satisfies

$$\|F(v, \pi, \eta)\|_{L^p(0,T;L^q(\mathcal{F}))} \leqslant C T^{\varepsilon}. \tag{5.20}$$

Using trace theorems, we deduce from (5.3) and from (5.10) that

$$\|\nabla v\|_{L^p(0,T;L^q(\partial \mathcal{F}))} \leqslant C, \quad \|v\|_{L^{3p/2}(0,T;L^{3q/2}(\partial \mathcal{F}))} \leqslant C T^{\varepsilon}.$$

From this relation, the above estimates and (5.9), (5.10), we deduce that H defined by (2.14) satisfies

$$\|H(v, \pi, \eta)\|_{L^p(0,T;L^q(\mathcal{S}))} \leqslant C T^{\varepsilon}. \tag{5.21}$$

Relations (5.20) and (5.21) yield that $\mathcal{N}(\mathcal{B}_{T,R}) \subset \mathcal{B}_{T,R}$ for T small enough. $\square$

Proposition 5.3. *With the above assumptions (in particular Assumption 5.1), there exists $T > 0$ small enough such that the map $\mathcal{N}_{T,R}$ (see (5.2)) is a strict contraction on $\mathbb{S}_{T,R}$.*

Proof. The proof is similar to the proof of Proposition 5.2, we only give the main ideas and omit the details. We consider $(f^{(i)}, h^{(i)})$, $i = 1, 2$. We have

$$\begin{aligned}\mathcal{N}_{T,R}(f^{(1)}, h^{(1)}) &- \mathcal{N}_{T,R}(f^{(2)}, h^{(2)}) \\ &= (F(v^{(1)}, \pi^{(1)}, \eta^{(1)}) - F(v^{(2)}, \pi^{(2)}, \eta^{(2)}), H(v^{(1)}, \pi^{(1)}, \eta^{(1)}) - H(v^{(2)}, \pi^{(2)}, \eta^{(2)})),\end{aligned} \tag{5.22}$$

where $(v^{(i)}, \pi^{(i)}, \eta^{(i)})$ is the solution to the system (4.2) in $(0, T) \times \mathcal{F}$ (see Corollary 4.7) associated with $(f^{(i)}, h^{(i)})$, $i = 1, 2$ and where F and H are given by (2.13)-(2.14). By taking T as Proposition 5.2, we have for each i that $(v^{(i)}, \pi^{(i)}, \eta^{(i)})$ satisfies the same property obtained in the proof of Proposition 5.2 and in particular, $X^{(i)}$, $Y^{(i)}$, $a^{(i)}$, $b^{(i)}$ defined by (2.6) and (2.11) satisfy also the same properties obtained in the proof of Proposition 5.2.

We write

$$v = v^{(1)} - v^{(2)}, \quad \pi = \pi^{(1)} - \pi^{(2)}, \quad \eta = \eta^{(1)} - \eta^{(2)}, \quad f = f^{(1)} - f^{(2)}, \quad g = g^{(1)} - g^{(2)},$$

Applying Corollary 4.7, we first obtain

$$\|v\|_{W^{1,2}_{p,q}((0,T);\mathcal{F})} + \|\pi\|_{L^p(0,T;W^{1,q}_m(\mathcal{F}))} + \|\eta\|_{W^{2,4}_{p,q}((0,T);\mathcal{S})} \leqslant C(\|f\|_{L^p(0,T;L^q(\mathcal{F}))} + \|h\|_{L^p(0,T;L^q(\mathcal{S}))}). \tag{5.23}$$

As in the proof of Proposition 5.2, the constants below may depend on R, but not on T and we assume $T \in (0,1)$ to simplify. Following the proof of (5.7), we can obtain

$$\|\eta\|_{L^\infty(0,T;C^1(\overline{\mathcal{S}}))} \leqslant CT^\varepsilon(\|f\|_{L^p(0,T;L^q(\mathcal{F}))} + \|h\|_{L^p(0,T;L^q(\mathcal{S}))}) \tag{5.24}$$

and following the proof of (5.8), (5.9) and (5.10), we deduce

$$\|v\|_{L^{3p}(0,T;L^{3q}(\mathcal{F}))} + \|\nabla_s^2\eta\|_{L^{3p}(0,T;L^{3q}(\mathcal{S}))} + \|\partial_t\eta\|_{L^{3p}(0,T;L^{3q}(\mathcal{S}))} \\ \leqslant CT^\varepsilon(\|f\|_{L^p(0,T;L^q(\mathcal{F}))} + \|h\|_{L^p(0,T;L^q(\mathcal{S}))}) \tag{5.25}$$

and

$$\|\nabla v\|_{L^{3p/2}(0,T;L^{3q/2}(\mathcal{F}))} + \|\nabla_s^3\eta\|_{L^{3p/2}(0,T;L^{3q/2}(\mathcal{S}))} + \|\nabla_s\partial_t\eta\|_{L^{3p/2}(0,T;L^{3q/2}(\mathcal{S}))} \\ \leqslant CT^\varepsilon(\|f\|_{L^p(0,T;L^q(\mathcal{F}))} + \|h\|_{L^p(0,T;L^q(\mathcal{S}))}). \tag{5.26}$$

Using trace theorems, we deduce from the above estimates that

$$\|\nabla v\|_{L^p(0,T;L^q(\partial\mathcal{F}))} \leqslant C(\|f\|_{L^p(0,T;L^q(\mathcal{F}))} + \|h\|_{L^p(0,T;L^q(\mathcal{S}))}),$$
$$\|v\|_{L^{3p/2}(0,T;L^{3q/2}(\partial\mathcal{F}))} \leqslant CT^\varepsilon(\|f\|_{L^p(0,T;L^q(\mathcal{F}))} + \|h\|_{L^p(0,T;L^q(\mathcal{S}))}).$$

We also deduce from the above estimate and from (2.6) that

$$\|\nabla X^{(1)} - \nabla X^{(2)}\|_{L^\infty(0,T;C^0(\overline{\mathcal{F}}))} + \|\nabla Y^{(1)}(X^{(1)}) - \nabla Y^{(2)}(X^{(2)})\|_{L^\infty(0,T;C^0(\overline{\mathcal{F}}))} \\ + \|a^{(1)}(X^{(1)}) - a^{(2)}(X^{(2)})\|_{L^\infty(0,T;C^0(\overline{\mathcal{F}}))} + \|b^{(1)} - b^{(2)}\|_{L^\infty(0,T;C^0(\overline{\mathcal{F}}))} \\ + \|\det(\nabla X^{(1)}) - \det(\nabla X^{(2)})\|_{L^\infty(0,T;C^0(\overline{\mathcal{F}}))} \leqslant CT^\varepsilon(\|f\|_{L^p(0,T;L^q(\mathcal{F}))} + \|h\|_{L^p(0,T;L^q(\mathcal{S}))}). \tag{5.27}$$

From (5.14) and from the above estimates, we obtain for all i,j,k,

$$\left|\frac{\partial a_{ik}^{(1)}}{\partial x_j}(X^{(1)}) - \frac{\partial a_{ik}^{(2)}}{\partial x_j}(X^{(2)})\right| \leqslant C|\nabla_s^2\eta| + CT^\varepsilon(\|f\|_{L^p(0,T;L^q(\mathcal{F}))} + \|h\|_{L^p(0,T;L^q(\mathcal{S}))}) \left(|\nabla_s^2\eta^{(1)}| + |\nabla_s^2\eta^{(2)}|\right), \tag{5.28}$$

$$\left|\frac{\partial^2 a_{ik}^{(1)}}{\partial x_j^2}(X^{(1)}) - \frac{\partial^2 a_{ik}^{(2)}}{\partial x_j^2}(X^{(2)})\right| \leqslant C\left(\left(|\nabla_s^2\eta^{(1)}| + |\nabla_s^2\eta^{(2)}|\right)|\nabla_s^2\eta| + |\nabla_s^3\eta|\right), \\ + CT^\varepsilon(\|f\|_{L^p(0,T;L^q(\mathcal{F}))} + \|h\|_{L^p(0,T;L^q(\mathcal{S}))}) \left(1 + |\nabla_s^2\eta^{(1)}|^2 + |\nabla_s^3\eta^{(1)}| + |\nabla_s^2\eta^{(2)}|^2 + |\nabla_s^3\eta^{(2)}|\right), \tag{5.29}$$

$$\left|\partial_t a^{(1)}(X^{(1)}) - \partial_t a^{(2)}(X^{(2)})\right| \leqslant C\left(|\nabla_s^2\eta| + |\nabla_s\partial_t\eta|\right) \\ + CT^\varepsilon(\|f\|_{L^p(0,T;L^q(\mathcal{F}))} + \|h\|_{L^p(0,T;L^q(\mathcal{S}))}) \left(1 + |\nabla_s^2\eta^{(1)}| + |\nabla_s\partial_t\eta^{(1)}|\right), \tag{5.30}$$

$$\left|\partial_t Y^{(1)}(X^{(1)}) - \partial_t Y^{(2)}(X^{(2)})\right| \leqslant C|\partial_t\eta| + CT^\varepsilon(\|f\|_{L^p(0,T;L^q(\mathcal{F}))} + \|h\|_{L^p(0,T;L^q(\mathcal{S}))}) \left(|\partial_t\eta^{(1)}| + |\partial_t\eta^{(2)}|\right), \tag{5.31}$$

$$\left|\frac{\partial^2 Y_\ell^{(1)}}{\partial x_j^2}(X^{(1)}) - \frac{\partial^2 Y_\ell^{(2)}}{\partial x_j^2}(X^{(2)})\right| \leqslant C|\nabla_s^2\eta| \\ + CT^\varepsilon(\|f\|_{L^p(0,T;L^q(\mathcal{F}))} + \|h\|_{L^p(0,T;L^q(\mathcal{S}))}) \left(1 + |\nabla_s^2\eta^{(1)}| + |\nabla_s^2\eta^{(2)}|\right). \tag{5.32}$$

Combining the above estimates with (5.11)–(5.19), with (5.8)–(5.10) and with (5.25)–(5.26), we deduce that

$$\left\|\mathcal{N}_{T,R}(f^{(1)},h^{(1)}) - \mathcal{N}_{T,R}(f^{(2)},h^{(2)})\right\|_{L^p(0,T;L^q(\mathcal{F}))\times L^p(0,T;L^q(\mathcal{S}))} \\ \leqslant CT^\varepsilon(\|f\|_{L^p(0,T;L^q(\mathcal{F}))} + \|h\|_{L^p(0,T;L^q(\mathcal{S}))}). \tag{5.33}$$

Thus for T small enough, we deduce the result. $\square$

6 Global in time existence

The aim of this section is to prove Theorem 1.2 and Theorem 2.3. The proof is similar to the proof of Theorem 1.1 and Theorem 2.2 given in Section 5. Throughout this section we assume the following

Assumption 6.1. *$(p,q)\in(1,\infty)$ satisfies (1.15) and (η_1^0,η_2^0,v^0) satisfies (2.18), (2.19), (2.20).*

Let us fix $\beta\in[0,\beta_0]$, where β_0 is introduced in Corollary 4.7 and for $R>0$, we define $\mathbb{S}_R$ as follows

$$\mathbb{S}_R := \left\{(f,h)\in L^p_\beta(0,\infty;L^q(\mathcal{F}))\times L^p_\beta(0,\infty;L^q(\mathcal{S}))\ ;\ \|f\|_{L^p_\beta(0,\infty;L^q(\mathcal{F}))}+\|h\|_{L^p_\beta(0,\infty;L^q(\mathcal{S}))}\leqslant R\right\}. \tag{6.1}$$

We take in what follows

$$R:=\|v^0\|_{B^{2(1-1/p)}_{q,p}(\mathcal{F})}+\|\eta_2^0\|_{B^{2(1-1/p)}_{q,p}(\mathcal{S})}+\|\eta_1^0\|_{B^{2(2-1/p)}_{q,p}(\mathcal{S})}$$

and to simplify the computation, we assume that $R\in(0,1)$.

In order to prove Theorem 2.3, we show that for R small, we can define the map

$$\mathcal{N}_R:\mathbb{S}_R\longrightarrow\mathbb{S}_R\quad (f,h)\longmapsto(F(v,\pi,\eta),H(v,\pi,\eta)), \tag{6.2}$$

where (v,π,η) is the solution to the system (4.2) in $(0,\infty)\times\mathcal{F}$ (see Corollary 4.7) and where F and H are given by (2.13)-(2.14). Then we show that for R small enough $\mathcal{N}_R(\mathbb{S}_R)\subset\mathbb{S}_R$ (see Proposition 6.2 below) and that, $\mathcal{N}_R|_{\mathbb{S}_R}$ is a strict contraction (see Proposition 6.3 below). This shows that $\mathcal{N}_R$ admits a unique fixed point and allows us to deduce Theorem 2.3.

First, we deduce from Corollary 4.7 that

$$\|v\|_{W^{1,2}_{p,q,\beta}((0,\infty);\mathcal{F})}+\|\pi\|_{L^p_\beta(0,\infty;W^{1,q}_m(\mathcal{F}))}+\|\eta\|_{W^{2,4}_{p,q,\beta}((0,\infty);\mathcal{S})}\leqslant CR. \tag{6.3}$$

By using [45, (7), p.196] and the Sobolev embeddings, we deduce from the above estimate

$$\|\eta\|_{L^\infty_\beta(0,\infty;C^1(\overline{\mathcal{S}}))}\leqslant CR. \tag{6.4}$$

Therefore, for R small enough, $\eta(t,\cdot)$ satisfies (2.1) for all $t\in[0,\infty)$ where c_0 is defined in (2.5). We can thus construct X by (2.6) so that $X(t,\cdot)$ is a C^1-diffeomorphism from $\mathcal{F}$ onto $\mathcal{F}(\eta(t))$. We can also consider $F(v,\pi,\eta)$ and $H(v,\pi,\eta))$ given by (2.13)-(2.14).

As in the previous section, we use (real or complex) interpolation results ([44, Theorem 2, p. 317]) to deduce that

$$\|v(t,\cdot)\|_{W^{s_2,q}(\mathcal{F})}\leqslant C\|v(t,\cdot)\|^{2/3}_{W^{s_1,q}(\mathcal{F})}\|v(t,\cdot)\|^{1/3}_{W^{2,q}(\mathcal{F})},$$

for any $s_2<2(1+s_1)/3$. Using (1.15), there exists $s_1\in(0,2(1-1/p))$ such that $s_2\geqslant 2/q$ so that by Sobolev embeddings,

$$\|v\|_{L^{3p}_\beta(0,\infty;L^{3q}(\mathcal{F}))}\leqslant C\|v\|^{2/3}_{L^\infty_\beta(0,\infty;W^{s_1,q}(\mathcal{F}))}\|v\|^{1/3}_{L^p_\beta(0,\infty;W^{2,q}(\mathcal{F}))}\leqslant CR. \tag{6.5}$$

and similarly,

$$\|\nabla_s^2\eta\|_{L^{3p}_\beta(0,\infty;L^{3q}(\mathcal{S}))}+\|\partial_t\eta\|_{L^{3p}_\beta(0,\infty;L^{3q}(\mathcal{S}))}\leqslant CR \tag{6.6}$$

and

$$\|\nabla v\|_{L^{3p/2}_\beta(0,\infty;L^{3q/2}(\mathcal{F}))}+\|\nabla_s^3\eta\|_{L^{3p/2}_\beta(0,\infty;L^{3q/2}(\mathcal{S}))}+\|\nabla_s\partial_t\eta\|_{L^{3p/2}_\beta(0,\infty;L^{3q/2}(\mathcal{S}))}\leqslant CR. \tag{6.7}$$

Using trace theorems, we deduce from (6.3) and from (6.7) that

$$\|\nabla v\|_{L^p_\beta(0,\infty;L^q(\partial\mathcal{F}))}\leqslant CR,\quad \|v\|_{L^{3p/2}_\beta(0,\infty;L^{3q/2}(\partial\mathcal{F}))}\leqslant CR. \tag{6.8}$$

We are now in position to prove the following result:

Proposition 6.2. *With the above assumptions (in particular Assumption 6.1), there exists $R > 0$ small enough such that the map $\mathcal{N}_R$ (see (6.2)) is well-defined and satisfies $\mathcal{N}_R(\mathbb{S}_R) \subset \mathbb{S}_R$.*

Proof. From (2.4) and (6.4), we deduce that for $T > 0$ small enough

$$\|\nabla X - I_3\|_{L^\infty_\beta(0,\infty;C^0(\overline{\mathcal{F}}))} + \|\nabla Y(X) - I_3\|_{L^\infty_\beta(0,\infty;C^0(\overline{\mathcal{F}}))} \\ + \|a(X) - I_3\|_{L^\infty_\beta(0,\infty;C^0(\overline{\mathcal{F}}))} + \|b - I_3\|_{L^\infty_\beta(0,\infty;C^0(\overline{\mathcal{F}}))} \leqslant CR, \quad (6.9)$$

$$\|\det(\nabla X) - 1\|_{L^\infty_\beta(0,\infty;C^0(\overline{\mathcal{F}}))} \leqslant CR, \quad \frac{1}{2} \leqslant \|\det \nabla X\|_{L^\infty_\beta(0,\infty;C^0(\overline{\mathcal{F}}))} \leqslant \frac{3}{2}, \tag{6.10}$$

$$\|\nabla X\|_{L^\infty_\beta(0,\infty;C^0(\overline{\mathcal{F}}))} + \|\nabla Y(X)\|_{L^\infty_\beta(0,\infty;C^0(\overline{\mathcal{F}}))} + \|a(X)\|_{L^\infty_\beta(0,\infty;C^0(\overline{\mathcal{F}}))} + \|b\|_{L^\infty_\beta(0,\infty;C^0(\overline{\mathcal{F}}))} \leqslant C. \tag{6.11}$$

We recall that a and b are defined by (2.11).

Using the above estimates, relations (5.15)–(5.19), (6.3), (6.5), (6.6), (6.7) and (6.8) we deduce that F and H defined by (2.13), (2.14) satisfy

$$\|F(v,\pi,\eta)\|_{L^p_\beta(0,\infty;L^q(\mathcal{F}))} + \|H(v,\pi,\eta)\|_{L^p_\beta(0,\infty;L^q(\mathcal{S}))} \leqslant CR^2, \tag{6.12}$$

which yields that $\mathcal{N}_R(\mathbb{S}_R) \subset \mathcal{S}_R$ for R small enough. □

We can also prove the following result by following the method used to prove Proposition 5.3 (we omit the proof).

Proposition 6.3. *With the above assumptions (in particular Assumption 6.1), there exists $R > 0$ small enough such that the map $\mathcal{N}_R$ (see (6.2)) is a strict contraction on $\mathbb{S}_R$.*

By combining Proposition 6.2 and Proposition 6.3, we deduce Theorem 2.3.

References

[1] R. A. Adams and J. J. F. Fournier, *Sobolev spaces*, vol. 140 of Pure and Applied Mathematics (Amsterdam), Elsevier/Academic Press, Amsterdam, second ed., 2003.

[2] H. Amann, *Linear and quasilinear parabolic problems. Vol. I*, vol. 89 of Monographs in Mathematics, Birkhäuser Boston, Inc., Boston, MA, 1995. Abstract linear theory.

[3] ———, *On the strong solvability of the Navier-Stokes equations*, J. Math. Fluid Mech., 2 (2000), pp. 16–98.

[4] ———, *Anisotropic function spaces and maximal regularity for parabolic problems. Part 1*, vol. 6 of Jindřich Nečas Center for Mathematical Modeling Lecture Notes, Matfyzpress, Prague, 2009. Function spaces.

[5] M. Badra and T. Takahashi, *Feedback boundary stabilization of 2D fluid-structure interaction systems*, Discrete Contin. Dyn. Syst., 37 (2017), pp. 2315–2373.

[6] ———, *Gevrey Regularity for a System Coupling the Navier–Stokes System with a Beam Equation*, SIAM J. Math. Anal., 51 (2019), pp. 4776–4814.

[7] H. Beirão da Veiga, *On the existence of strong solutions to a coupled fluid-structure evolution problem*, J. Math. Fluid Mech., 6 (2004), pp. 21–52.

[8] M. Bravin, *Energy equality and uniqueness of weak solutions of a "viscous incompressible fluid + rigid body" system with Navier slip-with-friction conditions in a 2D bounded domain*, J. Math. Fluid Mech., 21 (2019), pp. Art. 23, 31.

[9] A. Chambolle, B. Desjardins, M. J. Esteban, and C. Grandmont, *Existence of weak solutions for the unsteady interaction of a viscous fluid with an elastic plate*, J. Math. Fluid Mech., 7 (2005), pp. 368–404.

[10] P. Clément and J. Prüss, *An operator-valued transference principle and maximal regularity on vector-valued L_p-spaces*, in Evolution equations and their applications in physical and life sciences (Bad Herrenalb, 1998), vol. 215 of Lecture Notes in Pure and Appl. Math., Dekker, New York, 2001, pp. 67–87.

[11] R. Denk, M. Hieber, and J. Prüss, *$\mathscr{R}$-boundedness, Fourier multipliers and problems of elliptic and parabolic type*, Mem. Amer. Math. Soc., 166 (2003), pp. viii+114.

[12] R. Denk and R. Schnaubelt, *A structurally damped plate equation with Dirichlet-Neumann boundary conditions*, J. Differential Equations, 259 (2015), pp. 1323–1353.

[13] G. Dore, *L^p regularity for abstract differential equations*, in Functional analysis and related topics, 1991 (Kyoto), vol. 1540 of Lecture Notes in Math., Springer, Berlin, 1993, pp. 25–38.

[14] K.-J. Engel and R. Nagel, *One-parameter semigroups for linear evolution equations*, vol. 194 of Graduate Texts in Mathematics, Springer-Verlag, New York, 2000. With contributions by S. Brendle, M. Campiti, T. Hahn, G. Metafune, G. Nickel, D. Pallara, C. Perazzoli, A. Rhandi, S. Romanelli and R. Schnaubelt.

[15] Y. Enomoto and Y. Shibata, *On the $\mathscr{R}$-sectoriality and the initial boundary value problem for the viscous compressible fluid flow*, Funkcial. Ekvac., 56 (2013), pp. 441–505.

[16] S. Ervedoza, M. Hillairet, and C. Lacave, *Long-time behavior for the two-dimensional motion of a disk in a viscous fluid*, Comm. Math. Phys., 329 (2014), pp. 325–382.

[17] D. Fujiwara and H. Morimoto, *An L_r-theorem of the Helmholtz decomposition of vector fields*, J. Fac. Sci. Univ. Tokyo Sect. IA Math., 24 (1977), pp. 685–700.

[18] M. Geissert, K. Götze, and M. Hieber, *L^p-theory for strong solutions to fluid-rigid body interaction in Newtonian and generalized Newtonian fluids*, Trans. Amer. Math. Soc., 365 (2013), pp. 1393–1439.

[19] M. Geissert, M. Hess, M. Hieber, C. Schwarz, and K. Stavrakidis, *Maximal L^p-L^q-estimates for the Stokes equation: a short proof of Solonnikov's theorem*, J. Math. Fluid Mech., 12 (2010), pp. 47–60.

[20] O. Glass and F. Sueur, *Uniqueness results for weak solutions of two-dimensional fluid-solid systems*, Arch. Ration. Mech. Anal., 218 (2015), pp. 907–944.

[21] C. Grandmont, *Existence of weak solutions for the unsteady interaction of a viscous fluid with an elastic plate*, SIAM J. Math. Anal., 40 (2008), pp. 716–737.

[22] C. Grandmont and M. Hillairet, *Existence of global strong solutions to a beam-fluid interaction system*, Arch. Ration. Mech. Anal., 220 (2016), pp. 1283–1333.

[23] C. Grandmont, M. Hillairet, and J. Lequeurre, *Existence of local strong solutions to fluid–beam and fluid–rod interaction systems*, in Annales de l'Institut Henri Poincaré C, Analyse non linéaire, vol. 36, Elsevier, 2019, pp. 1105–1149.

[24] C. Grandmont, M. Lukáčová-Medvid'ová, and v. Nečasová, *Mathematical and numerical analysis of some FSI problems*, in Fluid-structure interaction and biomedical applications, Adv. Math. Fluid Mech., Birkhäuser/Springer, Basel, 2014, pp. 1–77.

[25] G. Guidoboni, M. Guidorzi, and M. Padula, *Continuous dependence on initial data in fluid-structure motions*, J. Math. Fluid Mech., 14 (2012), pp. 1–32.

[26] M. Guidorzi, M. Padula, and P. I. Plotnikov, *Hopf solutions to a fluid-elastic interaction model*, Math. Models Methods Appl. Sci., 18 (2008), pp. 215–269.

[27] B. H. Haak, D. Maity, T. Takahashi, and M. Tucsnak, *Mathematical analysis of the motion of a rigid body in a compressible navier–stokes–fourier fluid*, Mathematische Nachrichten, 292 (2019), pp. 1972–2017.

[28] M. Hieber and M. Murata, *The L^p-approach to the fluid-rigid body interaction problem for compressible fluids*, Evol. Equ. Control Theory, 4 (2015), pp. 69–87.

[29] P. C. Kunstmann and L. Weis, *Perturbation theorems for maximal L_p−-regularity*, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4), 30 (2001), pp. 415–435.

[30] P. C. Kunstmann and L. Weis, *Maximal L_p-regularity for parabolic equations, Fourier multiplier theorems and H^∞-functional calculus*, in Functional analytic methods for evolution equations, vol. 1855 of Lecture Notes in Math., Springer, Berlin, 2004, pp. 65–311.

[31] C. Lacave and T. Takahashi, *Small moving rigid body into a viscous incompressible fluid*, Arch. Ration. Mech. Anal., 223 (2017), pp. 1307–1335.

[32] J. Lequeurre, *Existence of strong solutions to a fluid-structure system*, SIAM J. Math. Anal., 43 (2011), pp. 389–410.

[33] J.-L. Lions and E. Magenes, *Problemi ai limiti non omogenei. V*, Ann. Scuola Norm Sup. Pisa (3), 16 (1962), pp. 1–44.

[34] D. Maity, J.-P. Raymond, and A. Roy, *Maximal-in-time existence and uniqueness of strong solution of a 3D fluid-structure interaction model.* working paper or preprint, Mar. 2018.

[35] D. Maity and M. Tucsnak, *A maximal regularity approach to the analysis of some particulate flows*, in Particles in flows, Adv. Math. Fluid Mech., Birkhäuser/Springer, Cham, 2017, pp. 1–75.

[36] D. Maity and M. Tucsnak, *L^p-L^q maximal regularity for some operators associated with linearized incompressible fluid-rigid body problems Selected Recent Results*, in Mathematical analysis in fluid mechanics selected recent results, vol. 710 of Contemp. Math., Amer. Math. Soc., Providence, RI, 2018, pp. 175–201.

[37] B. Muha and S. Canić, *Existence of a weak solution to a nonlinear fluid-structure interaction problem modeling the flow of an incompressible, viscous fluid in a cylinder with deformable walls*, Arch. Ration. Mech. Anal., 207 (2013), pp. 919–968.

[38] A. Quarteroni, M. Tuveri, and A. Veneziani, *Computational vascular fluid dynamics: problems, models and methods*, Computing and Visualization in Science, 2 (2000), pp. 163–197.

[39] J.-P. Raymond, *Stokes and Navier-Stokes equations with nonhomogeneous boundary conditions*, Ann. Inst. H. Poincaré Anal. Non Linéaire, 24 (2007), pp. 921–951.

[40] J.-P. Raymond, *Feedback stabilization of a fluid-structure model*, SIAM J. Control Optim., 48 (2010), pp. 5398–5443.

[41] ——, *Feedback stabilization of a fluid-structure model*, SIAM J. Control Optim., 48 (2010), pp. 5398–5443.

[42] T. Runst and W. Sickel, *Sobolev spaces of fractional order, Nemytskij operators, and nonlinear partial differential equations*, vol. 3 of De Gruyter Series in Nonlinear Analysis and Applications, Walter de Gruyter & Co., Berlin, 1996.

[43] R. Temam, *Navier-Stokes equations*, vol. 2 of Studies in Mathematics and its Applications, North-Holland Publishing Co., Amsterdam-New York, revised ed., 1979. Theory and numerical analysis, With an appendix by F. Thomasset.

[44] H. Triebel, *Interpolation theory, function spaces, differential operators*, Johann Ambrosius Barth, Heidelberg, second ed., 1995.

[45] ———, *Theory of function spaces*, Modern Birkhäuser Classics, Birkhäuser/Springer Basel AG, Basel, 2010. Reprint of 1983 edition [MR0730762], Also published in 1983 by Birkhäuser Verlag [MR0781540].

[46] L. Weis, *Operator-valued Fourier multiplier theorems and maximal L_p-regularity*, Math. Ann., 319 (2001), pp. 735–758.